A Theory for Prosocial Public-Housing Assignment: Finding Utility in the Translational Neuroscience from a Rodent Model

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Abstract

Meeting the challenges of promoting prosocial behavior in a public-housing complex through the optimal assignment of residents is a design goal of the public sector. Tenants of public-housing are often denied the opportunity to express choice when receiving their resident allocation. Not only can this process result in suboptimal allocations, but in the absence of considering the neurobehavioral implications of living communities, the approach may lead to a community with a predisposition to antisocial behavior. With a growing adaptation of cross-disciplinary connections, any new theory of public-housing assignment is obligated to consider neurobehavioral research contributions that explore how social stimuli promote prosocial and feeling states. Translational neuroscience acts becomes the innovation bridge by providing a translation of data and knowledge from fundamental animal-based neuroscience to an explanation of human brain function. This paper incorporates three distinct objectives. First the paper explores a theory for achieving an optimal public-housing structure to promote prosocial behavior. Second, the research presents a translation of a neurobehavioral animal study. The animal model investigates rearing environments as way to proxy the public-housing community. The resultant translation produces a standards-based per capita (pro-) social indicator score (SIS). Third, based on a hypothetical SIS metric, a basic multiple-objective resident eligible model for public-housing assignment is proffered. The contribution of this interdisciplinary and translational theory is to promote a mathematical theory that incorporates translation neuroscience for the purpose of optimizing prosocial outcomes in public-housing apartment assignment within the ‘smart’ city.

Keywords: OR in Government; OR in societal problem analysis; Community operational research.

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A measurable result of the 1929 Great Depression and the recession of 1937-38 was a weakening of the U.S. urban municipal core. Owing to these economic events, depressed urban areas experienced a deterioration of existing housing stock and, where new building did take place, haphazard housing development became the norm. This socio-economic reality led the U.S. Congress to pass the 1937 Housing Act; a law that provided the legal foundation of subsidized local public-housing. The goal was to provide capital to local agencies for the purpose of improving unsafe housing conditions and to alleviate acute shortages of decent and safe dwellings for low-income families. But, the arrival of the 21st century finds, for many urban communities, these goals remain unmet. More specifically, within urban housing-projects, the process of evolving social identity as a natural regulator of prosocial behavior and housing security has stalled. The counterpart, antisocial behavior, not only remains embedded in housing-project communities but, in some cases, this behavior appears to have overtaken prosocial goal achievement. For cities seeking to enter or expand 'smart' city status, this condition can threaten new urban growth built on a framework of safe housing alternatives across all economic strata. In fact, antisocial behavior is a recognized factor for leading urban youth into stressful conditions; conditions which often result in a pathway to organized family services (Park, Lee, Sun, Vazsonyi, & Bolland, 2010). Aided by the sheer scale of the urban environment, a size that can foster stress associated with the feeling of isolation, antisocial behaviors are completely disruptive to the associative benefits of residential trust behavior[1].

Without argument, the efficacy of the public sector depends on how well service delivery is designed. As cities seek 'smart' status around the desirable strategy of investing in electronic data collection and distribution, urban managers must also recognize that eliminating suboptimal public-housing apartment assignments is a realistic path towards improving resident welfare gains (Thakral, 2016). This design path requires new thinking and a new approach to the vexing problem of how to best achieve prosocial societal evolution in public-housing. To that end, the fully 'smart' city will materialize only when contemporary housing science embraces a synergy across the social sciences, neuroscience, and operational research.

Extant literature provides a history of how translational neuroscience has relied on the physiology of animal models to evolve practical decision models to enhance human well-being. For example, the research strain pioneered by the "rat cities" studies of Calhoun (1952), the "fight or flight" studies of Christian (1950; 1961), and the "welfare state" study of Richter (1959) provide the historical link. Collectively, these extant findings on crowding pathology and the existence of a behavioral sink

[1] Examples of residential trust behavior are: social foraging, access to mating equitable partners, group vigilance and increased growth in income and economic development (Gould & Hijzen, 2016)

(huddling in knots of squalor and violence) led researchers to postulate the existence of a threshold point where rats became overwhelmed by the crowding state. By translation, the message was that crowding results in pathological behavior in rats and by extension to humans. What was left unanswered by these earlier studies was a definitive understanding of sex differentials.

Until recent edicts by the National Institutes of Health (NIH), female mammals were neglected in biomedical research leading to a failure to investigate and analyze data by sex (Beery & Zucker, 2011). By way of example, and to a measureable extent, male and female rodents are socialized differently by rearing environment. In contrast to an isolated environment (IE)[2], a social environment (SE)[3] or an enriched environment (EE)[4] generally protect animals against stress. Published investigations of defective social experience report how CD-1 mice (outbred Swiss mice) exposed to enriched and social environments displayed lower locomotion and anxiety-like responses compared to those housed in isolated environments (Ros-Simó & Valverde, 2012). Rats reared in an EE were also reported to exhibit enhanced motor, sensory, social, and cognitive functions (Urakawa, Mitsushima, Shimozuru, Sakuma, & Kondo, 2014). Fear learning of rats is also differentiated by rearing environment where it is noteworthy that "observational fear" between adolescent rates (not acquired fear) is affected by SE versus EE (Panksepp & Lahvis, 2016).

When controlling for biological sex within environment, research reports that social housing reduces stress and corticosterone levels in female rats but not males (Brown & Grunberg, 1995). Sex differences are also reported for the fear/anxiety neural circuitry. Compared to males, blunted neural activations for female rats exposed to social environments, or anxiogenic stimuli, is evidenced by lower immediate early-gene, *c-fos* immunoreactive (Fos-IR or Fos) cells in the amygdala and paraventricular nucleus (Trentani et al., 2003). Separately, female rats also displayed lower *c-fos* counts in the paraventricular nucleus of the hypothalamus in response to chronic stress and social housing relative to males (Westenbroek, den Boer, & Horst, 2003). As rodent-based studies advanced over time, the accumulated evidence on rat neural activation gained support in translational neuroscience and clinical practice (Pober, Neuhauser, & Pober, 2001). More recently, the relevance of translational neurobehavioral rat studies were strengthened by findings that rats have a counterpart to every known human gene associated with disease (Shimoyama et al., 2016) and that different sensory inputs can cause motor commands to move a rat's whiskers – an indication of complex neural process that are consistent with human observation (A. B. Smith & Alloway,

[2] In an isolated environment a single rat is placed in a cage with a metal cage top and housed one rate with no physical contact with other rats.
[3] A social environment consists of a larger cage with a more pine chips, and is covered with a wire mesh. It housed same-sex/phenotype animals in groups.
[4] An enriched environment is by even larger cages with toys, tunnels, ladders, running wheels and a few same-sex mates.

2013).

This paper has three central objectives. The first is to present a theory for modeling public-housing assignment. The second is to abstract a neurobehavioral study on Long-Evans rats to develop a framework for computing a social indicator score (SIS) that can be translated to individual responses of social, economic and psychological welfare (Saris, van Wijk, & Scherpenzeel, 1998). A third contribution of the paper is the presentation of a baseline multicriteria and multiple-objective decision model (MCDM) for public-housing assignment to allocated apartments. In the MCDM model, eligible resident applicants are assigned by a human SIS which is computed by translation of rodent behavior from the study of trait and sex in alternate rearing environments. On a per capita basis the translated SIS is a proxy for individual behavioral strategies and is incorporated in the MCDM of public-housing assignment. We complete the framework by investigating a baseline MCDM to demonstrate one characterization of the prosocial assignment model and its design of how best to promote 'smart' housing science when managers are faced with an array of demographic changes[5]. Given the premise that social stimulation of neural circuits influence either prosocial or antisocial actions and feeling states (Numan, 2015), the research begins by recasting the well-known theory on pollution and city size as an economic theory of prosocial residential assignment and behavior[6].

A Theory of Antisocial Behaviors and Public-Housing

The generalized monocentric housing assignment model presented below assumes the residents of the public-housing community work, consume goods, and are uniformly impacted by a unique set of externalities. The model recognizes, but does not address, spillover effects across alternate communities or across cities.

Model with Symmetric Buildings

Within city bounds, the symmetric building model assumes there are M apartments available of which m are homogeneous government subsidized apartments located across I dispersed heterogeneous buildings. The population seeking apartments within city bounds is exogenous and denoted by N. Hence, it is inferred that the subset of eligible tenants for public-housing is $n < N$. In the SMART city, it is convenient to refer to city-wide economic and social-sustaining opportunities as activities which occur in the central business district (CBD). Our model assumes that n residents experience well-defined commuting times to obtain social service support and participation in employment alternatives. Under this structure, eligible residents have identical preference functions given by

[5] These changes include: the non-standard family unit; cultural shifts (new paradigms of the sharing economy); new levels of density (sustainable mixed-use property); and, economic transmutations.

[6] To fully understand the intent of our recast, we advise the reader to consult the sourced economic reasoning. Specifically, see Borck and Tabuchi (2016).

$$u(s,z,E) = s^{\alpha} z^{1-\alpha} E^{-\beta} \tag{1}$$

with the corresponding budget constraint

$$w = z + rs + td \tag{2}$$

In this system of equations, s expresses the utility of a desired increase in housing consumption relative to the non-housing basket of goods, z. City-wide antisocial behavior is captured by E. Wage income is expressed by w and r is the market housing rent per unit of measure for assigned floor space. The cost of travel, t, is with respect to distance, d, to the CBD. We note that $0 < \alpha < 1$, and $\beta > 0$. In the uni-objective specification families seek an apartment unit (floor space) by choosing s and z to maximize (1) subject to (2). That is, for eligible residents the optimal consumption of an apartment unit is stated as

$$s(w-td, r) = \frac{\alpha(w-td)}{r} \tag{3}$$

Eligible residents who are seeking assignment to an apartment are mobile within and across the I buildings. Dwellings are priced to reflect the highest bid permitted by the managing government authority. We determine the maximum amount the eligible household is willing to pay for an apartment unit in a given building by substituting (3) and (2) in (1) and solving $u(z,s,E) = \bar{u}$ resulting in

$$r(w-td, E, v) = (w-td)^{1/\alpha} E^{-\beta/\alpha} v^{-1/\alpha} \tag{4}$$

where $v \equiv \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \bar{u}$.

The model's equilibrium conditions for the city-wide apartment assignment are:

$$r(w-t\bar{d}, E, v) = r_A \tag{5}$$

$$\int_0^{\bar{x}} \frac{1}{s(w-td, E, v)} dx = n \tag{6}$$

In the first condition, Eq. (5), the term r_A is the Ricardian agricultural land rent for an apartment. The second condition, Eq. (6), states that the eligible tenant population, n, fills available apartment units between 0 and $\bar{x}$.

Now suppose external economies of scale exist at the building level. Scale advantages accrue from group vigilance (security and safety enhancements), partner matching, and other recognized manifestations of trust behavior. The building's local production is assumed to be $Y = n^{1+\gamma}$, with $0 < \gamma < \alpha$ and the individual wage is $w = n^{\gamma}$ (Duranton & Puga, 2004). By substituting (3) and (4) into (5) and (6) we derive the functional border of city-wide public-housing apartments and an associated indirect utility.

$$\bar{x} = \frac{n^{\gamma}\left[1 - r_A^{\alpha}\left(r_A + tn\right)^{-\alpha}\right]}{t} \tag{7}$$

$$v = n^{\gamma}\left(r_A + tn\right)^{-\alpha} E^{-\beta} \tag{8}$$

Equation (7) shows that as the eligible population increases, a building's vacant apartments, $\bar{x}$, are assigned and filled. In this model, separable utility infers that $\bar{d}$ is not affected by antisocial behavior. Equation (8) accounts for the well-known agglomeration effect. The agglomeration effect increases utility with increasing n. But, as the increasing number of eligible residents migrate throughout the city, the possibility of increasing commute times, *d*, to the CBD along with higher land rents due to increasing competition for an apartment assignment results in a decrease in utility. In the next section, we examine the impact of antisocial behavior on this tradeoff.

Antisocial Behavior

Antisocial behavior in the *i-th* public building is given by

$$E_i = e\left(n_i\right) + \delta \sum_{j=1}^{m_i - 1} e\left(n_j\right) \tag{9}$$

where *e* is the baseline antisocial behavior for the building and $0 \le \delta \le 1$ measures the aggregated degree of building per capita antisocial spillovers to other buildings emanating from the unique characteristics of residents assigned to apartments in a specific building. Thus $\delta = 0$ implies antisocial behavior is restricted to a unit building. Conversely $\delta = 1$ implies antisocial behavior is endemic to all public-housing buildings within the city bounds.

Equilibrium and Optimum Number and Size of Urban Projects

In a SMART city with an evolving symmetric public-housing building architecture, a prosocial equilibrium is defined by the solution of $v_i = v^*$ for all *i*. We also note that if $v\left(0\right) = 0$ then no one would elect to live in a government controlled public-housing building. This begs the question – what is the impact of migration when all antisocial behavior is localized at the building level? In this case, migration is defined by the utility differential

$$v\left(n_i\right) - v\left(n_j\right) = v\left(n_i\right) e\left(n_i\right)^{-\beta} - \hat{v}\left(n_j\right) e\left(n_j\right)^{-\beta} \tag{10}$$

where $\hat{v}\left(n_j\right) := n^{\gamma}\left(r_A + tn\right)^{-\alpha}$.

Also, for the *i-th* building, we assume antisocial behavior $e(n_i)$ satisfies $e(0) = 0$ and $de / dn_i > 0$.

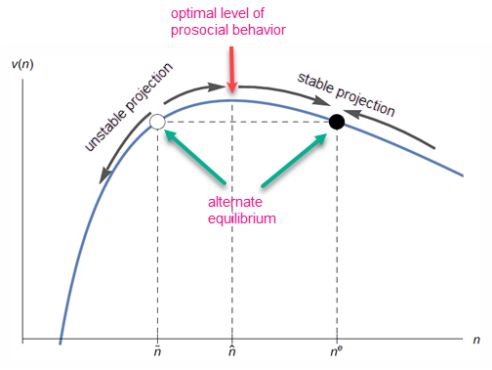


Figure 1. Optimal Prosocial Behavior for a Building

Figure 1 above shows the optimal level of prosocial behavior, $\hat{n}$, for any building in the city along with two alternate equilibrium levels, $\tilde{n}$ and n^e. Any prosocial equilibrium $\tilde{n} < n^e$ results in an unstable projection of prosocial behavior. This result implies that any deviation from an equilibrium prosocial state would result in migration out of an unstable building. Conversely, as indicated by the arrows, any equilibrium $n^e > \tilde{n}$ is a prosocial stable condition for an individual urban apartment building. A deviation from n^e would induce migration flows to restore equilibrium. There is a continuum of equilibria with $n^e > \hat{n}$ where $\hat{n} = n^*$ such that it maximizes $v(n_i)$. We summarize this as:

Proposition 1. *If antisocial behavior is purely localized by building with no possibility of spillover effects, $\delta = 0$, then in equilibrium overall building antisocial behavior is measured by how a change from an over- or under-populated count in eligible residents impacts the per capita contribution to antisocial outcomes within an individual building.*

Global Antisocial Behavior

The question arises about how eligible residents' value apartment assignment when there is a widespread belief that antisocial behavior is globally endemic to all buildings hosting government subsidized apartments. We examine this effect by letting $\delta = 1$. Under this condition, it is not possible for eligible residents seeking assignment to differentiate antisocial behavior at the building level. Without this differentiation, we can drop the index *i* from building specific antisocial behavior (E_i). We write the utility difference of living in building *i* versus building *j* as:

$$v(n_i) - v(n_j) = E^{-\beta}\left(\hat{v}(n_i) - \hat{v}(n_j)\right) \tag{11}$$

When $E > 0$, eligible residents decide against apartment migration since the migration decision is based on the difference $\hat{v}(n_i) - \hat{v}(n_j)$. Let $\hat{n}$ denote a building's capacity, which solves to $Max_n \hat{v}(n)$. Setting $v'(n) = 0$ gives

$$\hat{n} = \frac{\gamma r_A}{(\alpha - \gamma)t} \tag{12}$$

For a continuum of stable equilibria with building sizes $n^e > \hat{n}$, we follow the arguments of Henderson (1974) to postulate that any public-housing building equilibrium with $n^e > \hat{n}$ is in a stable prosocial state. The optimum level of eligible resident occupancy to sustain building-wide prosocial behavior, n^*, is found by maximizing $v'(n) = \hat{v}(n)E(n)^{-\beta}$. The first order condition is written accordingly,

$$\hat{v}'(n) - \beta \hat{v}(n)\frac{E'(n)}{E(n)} = 0 \tag{13}$$

We know that $n^e \geq \hat{n}$ and that $\hat{n}$ maximizes $\hat{v}(n)$. Since $\beta > 0, E(n) > 0$ and $\hat{v}(n)$ is quasi-concave, evaluating (12) at $\hat{n}$ implies that $n^* < \hat{n}$ **if** $E'(\hat{n}) > 0$. Since $E(n) = m \cdot e(n) = \frac{n}{m} e(n)$, if resident antisocial behavior increases with the number of assignments made to a building then the residential antisocial target for the building either exceeds recommended limits or some apartments house undocumented tenants. When this is the case then any increase in building density increases the disutility from antisocial behavior. On the other hand, if prosocial behavior is increasing with the expansion of building density, we find $n^* > \hat{n}$. In equilibrium, this opens the possibility for increasing building density until $n^e > \hat{n}$.

Proposition 2. *Suppose that antisocial behavior is global to the city-wide building density, i.e., $\delta = 1$. If eligible resident antisocial behavior increases with n, then city-wide public-housing density is beyond established occupancy guidelines or ineligible residents are providing covinous housing. However, if per capita antisocial behavior decreases with a change of n, then building density may either be too small or too large in equilibrium.*

The theoretical model presented above has a certain normative appeal for economies with public goods. The model provides a framework for efficient apartment assignment where all available subsidized residential alternatives are indistinguishable to the population of eligible residents. Hence, achieving optimal prosociality in apartment mix is a goal that can be independently defined within the context of articulating a broader set of public objectives.

The Translational Rodent Model

The translational aim of this study requires a rat experiment to study *c-fos* expression as a marker for neuronal activity following stimulation. As a proto-oncogene, *c-fos,* is identifiable by immunohistochemical techniques. It is expressed within some neurons following depolarization. In our previous work, Liu et. al (2018), we examined sex based behavioral and physiological aspects of social housing in trait anxiety rats. The experiment was conducted on adult filial-3 (F3) 4th generation Long-

Evans rats (*n*=80) which were selected from an outbred line of extreme anxiety rats at the University of Massachusetts Boston. We summarize the Liu et. al (2018) experiment in Figure 2. This figure shows the husbandry separation which resulted in eight experimental groups for the three fixed effects of trait, environment and sex. Trait separation in the study was binary between high anxiety (HAn) and low anxiety (LAn) animals. Environment represents either the isolated rearing environment (IE) or the social environment (SE).

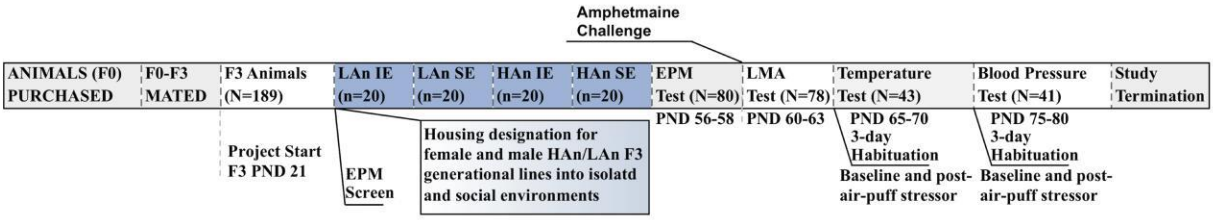


Figure 2. Schematic Diagram and Timeline of the Animal Experiment

Post 35-day habituations in either the IE or SE environment, sequential assays were applied to the subjects. *In vivo*, we examined how rearing environment and biological sex interacted to modulate neural markers of anxiety-like behavior across the entire study group. The behavioral assays were executed using an elevated plus maze (EPM) and a locomotor activity (LMA) chamber. Supportive findings factored into this study are:

- Sex and Trait within Environment reveal a positive modulating effect of SE for both female and male rats and specifically in the HAn lines.
- Increased drug (amphetamine) induced locomotion in HAn rats began between 10-15 minutes post-injection and was sustained for 35 to 40 minutes. By contrast, the LAn rats showed a delayed (40-minutes post-injection) increase in locomotion. HAn females exhibited the fastest locomotor activation and sustained drug sensitivity in distance traveled.
- Trait significantly explained rodent high blood-pressure whereas social housing contributed to a reduction in diastolic blood pressure – especially for females.
- Only HAn male rats benefit from a social rearing environment.

Following the acute exposure to the open field novel stress test[7] in the LMA and a dose of amphetamine (AMPH) four rats, chosen at random from each of the eight experimental groups (*n*=32), were anesthetized with an overdose of sodium pentobarbital (1ml/0.1kg). We seek to confirm that sex and environment tend to mediate both trait and drug-induced hyperactivity (Hammerslag, Waldman, & Gulleya, 2014). Specifically, our aim is to measure neural activation along the reward and fear circuitry

[7] Open field stress test is an experimental test used to assay general locomotor activity levels, anxiety, and willingness to explore in animals. It is conducted in the LMA chamber.

by capturing *c-fos* levels in seven left-and-right brain regions, respectively. The 32 rats were transcardially perfused with isotonic (0.9%) saline followed by 4% paraformaldehyde. Following Paxinos and Watson (2004), the brains were removed and stored in the fixative and later blocked for reward (+4.0 to 0 Bregma) and fear (0 to -4.0 Bregma) circuitry areas. The sections were cryoprotected 48 hours prior to microsectioning in 10% sucrose- (Day 1) and then 20% sucrose-4%paraformaldehyde solution (Day 2). Coronal sections (30μm) were collected and stored at -80°C in a cryoprotectant until time of immunohistochemistry.

Immunohistochemistry. Immunohistochemistry was run on free-floating sections rinsing with 0.05M sodium phosphate buffer saline (NaPBS) at pH 7.4 and incubated in anti- *c-fos* IgG (1:8000, Santa Cruz Biotechnology, CA) for 1 hour at room temperature and then at 4°C for 48 hours. Next, the sections were rinsed multiple times in biotinylated goat anti-rabbit (1:600, Vectastain, Vector Laboratories Ltd), followed by the avidin-biotin complex for 1-hour. The tissue underwent a final flush with Tris-buffered saline (TBS), were incubated in 3,3'-diaminobenzidine (DAB) solution (DAB Peroxidase Substrate Kit, Vector Laboratories) for amplifying the stain, and subsequently placed into reverse osmosis H_2O. Finally they were mounted on gelatin coated slides for viewing the Fos immunoreactive cells using light microscopy.

Fos Immunoreactive cell counts. Within the *FOS* family of genes, the immediate early gene, *c-fos,* peaks within 60 minutes following a behavioral experience. To document *c-fos* changes, blind observers used a SPOT Flex monochrome camera to capture three bilateral images of seven brain regions. Fos immunoreactive cells were counted within a 250 μm^2 area in each image at 20× magnification, using ImageJ software from NIH. The brain images were taken in regions implicated by: a) stimulant drugs and social stress (periaqueductal gray (PAG)); b) fear/anxiety – defensive emotion (ventromedial hypothalamus (VMH)); c) fear/anxiety – development of fight or flight emotion (amygdala (Amy)); d) fear/reward (nucleus accumbens core NAc(c) and shell NAc(sh)); and, e) cognitive control of emotion – cingulate gyrus 1 (Cg1) and cingulate gyrus 2 (Cg2). Subsequently, *c-fos* cell counts were averaged across the three images. This statistical aggregation yielded 14 average *c-fos* cell counts for each of the 32 rodents.

Data Analysis

Statistical analyses were performed using SPSS (v. 22.0). The *a priori* level of significance for all parametric tests was set at 0.05. For the 14 average *c-fos* cell counts per rat we computed descriptive statistics and an exploratory factor analysis (EFA). The latter test was conducted to confirm orthoganllity of the anxiety and stress latent dimensions. Immediately, we invoke a non-parametric multivariate radial basis function artificial neural network model (MRANN) to test the hypothesis that there is a significant implication of *c-fos* due to the independent variables (i.e. sex, trait, and environment). For the MRANN

procedure we implemented split observations for training and validation (70 / 30, respectively). We choose an exponential transfer function for the hidden layer. The synaptic weights were estimated by gradient descent.

Results

Descriptive Statistics

Table 1presents the mean and standard deviation for the average *c-fos* counts. Male rodents' *c-fos* counts are primarily in the third and fourth quartiles for PAG, Amy, Cg1 and Cg2 regions (columns 1-8). The exception is the LAn SE male group. This group's mean *c-fos* counts were below the median in Cg2 (column 7-8) and Cg1R (column 6). The HAn SE male group exhibited *c-fos* readings above the median, a finding that extends to all regions except the VMH (column 13-14). In the male groups, all except the HAn IE group had a high reaction to AMPH and stress. Additionally, and unlike their female counterparts, all male groups had high *c-fos* readings in the fear / anxiety circuitry. The LAn IE males were the only group to exhibit mean *c-fos* readings in the fourth quartile for Cg-1. Lastly, we report that HAn SE males exhibited high *c-fos* counts in the fear / reward circuitry relative the entire group of rats.

Regardless of trait and environment, the female group exhibited fourth quartile results for *c-fos* means primarily in the PAG area (column 1-2). The HAn IE female group also showed a relatively high mean in the VMH region (column 13-14) with a more moderate mean level in the Amy region (column 3-4). Females generally demonstrate a greater immediate early gene reaction to AMPH and stress. HAn IE females also displayed a higher level of fear circuitry activation following the acute stress.

Table 2, presents the Pearson correlation coefficients (ρ) for the average *c-fos* positive cells. As expected, the left- and right-side of the brain regions exhibit significant correlation coefficients ($p \leq 0.05$) in all experimental groups.

Table 1. *Mean and Standard Deviations of Average c-fos Counts from Left- and Right-Brain Regions of Male and Female High and Low Anxiety Rats in Different Housing Environments*

		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Female															
LAn IE	M	*11.75*	**13.33**	9.42	10.17	9.75	9.08	9.75	9.34	9.75	9.08	9.75	9.34	9.08	6.00
	SD	1.29	4.12	7.65	8.31	3.01	1.66	3.89	5.10	3.01	1.66	3.89	5.10	6.38	2.45
LAn SE	M	**16.42**	**22.00**	8.33	*11.00*	*12.59*	10.67	*11.59*	10.42	*12.59*	10.67	10.42	10.42	*10.75*	5.75
	SD	4.14	5.59	6.70	8.04	2.36	2.60	2.46	3.36	2.36	2.60	3.36	3.36	8.21	4.92
HAn IE	M	**13.42**	**15.00**	*12.42*	*11.08*	7.92	6.83	7.25	8.00	7.92	6.83	7.25	8.00	**14.09**	**15.25**
	SD	8.33	9.76	7.55	8.85	1.69	2.73	1.97	3.23	1.69	2.73	1.97	3.23	2.53	4.03
HAn SE	M	**14.33**	**17.58**	8.58	7.50	*10.75*	7.92	7.08	**13.17**	*10.75*	7.92	7.08	**13.17**	7.58	8.50
	SD	3.89	5.07	2.13	2.12	6.60	3.63	1.20	15.59*	6.60	3.63	1.20	15.59*	3.83	3.87
Male															
LAn IE	M	**14.83**	**15.92**	**17.17**	**20.58**	**24.00**	**12.59**	**10.25**	**13.58**	6.75	*11.42*	10.17	8.75	10.00	*11.08*
	SD	7.04	4.58	6.89	4.63	8.74	1.26	2.28	5.07	0.83	2.32	3.25	3.76	8.66	7.88
LAn SE	M	*10.67*	**13.17**	**15.00**	**14.25**	*11.33*	7.67	8.58	7.75	8.33	9.42	8.42	9.50	*12.17*	8.25
	SD	0.86	3.13	6.81	4.58	4.74	1.96	2.64	2.75	0.67	3.01	2.54	4.99	10.46	1.60
HAn IE	M	10.25	9.50	**19.83**	**18.17**	*11.17*	*12.92*	*11.25*	*12.58*	*7.67*	10.42	7.33	*11.34*	8.34	9.08
	SD	3.86	4.24	8.40	5.56	2.38	4.59	5.88	6.99	3.39	9.18	2.79	6.42	5.13	2.47
HAn SE	M	**12.08**	**15.92**	**16.25**	**13.17**	**13.92**	*10.83*	*11.75*	**19.42**	**18.33**	**13.25**	**15.00**	**17.67**	6.58	8.67
	SD	2.42	6.79	5.85	6.25	7.33	5.31	7.38	19.06*	18.63*	15.44*	15.34*	17.94*	2.76	1.25

Note: Where 1 = PAG-L; 2 = PAG-R; 3 = Amy-L; 4 = Amy-R; 5 = Cg1-L; 6 = Cg1-R; 7 = Cg2-L; 8 = Cg2-R; 9 = NAc(c)-L; 10 = NAc(c)-R; 11 = NAc(sh)-L; 12 = NAc(sh)-R; 13 = VMH-L; 14 = VMH-R. Boldface indicates means in the fourth quartile. Means in the third quartile are italicized. *Standard deviations ≥ 15.0.

Table 2. *Intercorrelations of c-fos Counts of Left- and Right-Brain Regions*

Regions	1	2	3	4	5	6	7	8	9	10	11	12	13
1. PAG-L	–												
2. PAG-R	.75**	–											
3. Amy-L	.12	-.04	–										
4. Amy-R	.10	-.06	.70**	–									
5. Cg1-L	-.10	-.12	.18	.28	–								
6. Cg1-R	-.04	-.11	.36*	.29	.60**	–							
7. Cg2-L	-.03	-.05	.20	-.01	.32	.50*	–						
8. Cg2-R	.06	.04	-.01	-.13	.21	.34	.55**	–					
9. NAc(C)-L	.01	.13	-.16	-.25	-.03	-.03	.58**	.75**	–				
10. NAc(C)-R	.02	.15	.08	-.04	.06	.23	.72**	.77*	.78**	–			
11. NAc(Sh)-L	.18	.20	-.09	-.14	-.01	.02	.71**	.65**	.86**	.85**	–		
12. NAc(Sh)-R	.03	.13	-.13	-.23	.05	.18	.46**	.93**	.78**	.79**	.67**	–	
13. VMH-L	.16	-.05	.16	.09	-.10	-.01	.04	-.03	-.07	.05	.09	-.04	–
14. VMH-R	.31	.01	.25	.07	-.13	.02	-.03	.15	-.06	.11	.09	.10	.60**

Note: The suffix of L and R signify the left and right brain regions, respectively. *$p < 0.05$. **$p < 0.01$.

Exploratory Factor Analysis (EFA)

To extract latent structure from the underlying correlation matrix (Table 2), an EFA with an orthogonal Varimax rotation (Kaiser Normalization method) was performed. Four neurobehavioral factors with eigenvalue ≥ 1.0 were retained. The factors accounted for 76% of the variation across average *c-fos* induction.

The factors were labeled by the brain regions that have high factor loadings ($\boldsymbol{\ell}$). Thus, factor 1 which was represented by NAc(c) ($\boldsymbol{\ell} \geq 0.90$), NAc(Sh) ($\boldsymbol{\ell} \geq 0.89$), and Cg2 ($\boldsymbol{\ell} \geq 0.75$) was labeled as the *'Fear / Reward Emotional Circuit'*. Both NAc and Cg2 have a significant role in cognitive reasoning, especially when it comes to reward, motivation, and reinforcement learning, and are also thought to integrate circuitry implicated in cognitive and emotional functioning (Allman, Hakeem, Erwin, Nimchinsky, & Hof, 2001). Factor 2 represented by Cg1 ($\boldsymbol{\ell} \geq 0.69$), and Amy ($\boldsymbol{\ell} \geq 0.73$) expresses the *'Fear / Anxiety Emotional Circuit'*. Both Cg1 and Amy play a significant role in episodic memory, decision-making and emotional reaction, especially to fear brought on by anxiety. Factor 3 labeled as '*Stimulant Drug and Social Stress Factor'* is dominated solely by PAG ($\boldsymbol{\ell} \geq 0.95$). Lastly, explanatory loadings on factor 4 are attributed to VMH ($\boldsymbol{\ell} \geq 0.86$). While VMH and Amy both play a significant role in the fear / anxiety circuitry, VMH plays an additional role by capturing the rat's defense mechanism. The factor is labeled the *'Fear/Anxiety Escape Circuit'*.

Derivation of a Rodent Model Social Indicator Score

Following Thomson (1951) and Kajiji and Dash (2011), we create factor score vectors from the EFA[8]. The per capita SIS is calculated as a simple average across the independent factor score vectors:

$$SIS_{i,n} = \delta_i \frac{\sum_{j=1}^{C} f_{i,j}}{C} \tag{14}$$

where f_i is the score of the *i-th* rat on the *j-th* latent dimension. Incorporating propositions 1 and 2 above, δ_i weights the SIS by individual spillover behavior. As an equation coefficient in an MCDM model, the SIS metric effectively conveys the per capita spillover adjusted mindset of rationality and emotion (Wenstop, 2005).

The MRANN Nonparametric analysis of Rodent Data

Figure 3 displays MRANN results for the PAG and Amy brain regions by way of example. For the PAG model, the MRANN design consisted of five hidden nodes. By contrast, within the Amy model, only two hidden nodes were required to populate the network. Although interaction effects are not

[8] We propose administering social-survey questionnaire(s) and anxiety-inducing tests with the goal of extracting quantifiable physiological and anxiety markers to describe all eligible applicants (Apoorvagiri & Nagananda, 2015) in a factor structure that mimics the EFA latent dimensions uncovered in the rat experiment.

explicitly reported in the MRANN output, we note that every possible network combination is enumerated within the computation of the hidden nodes.

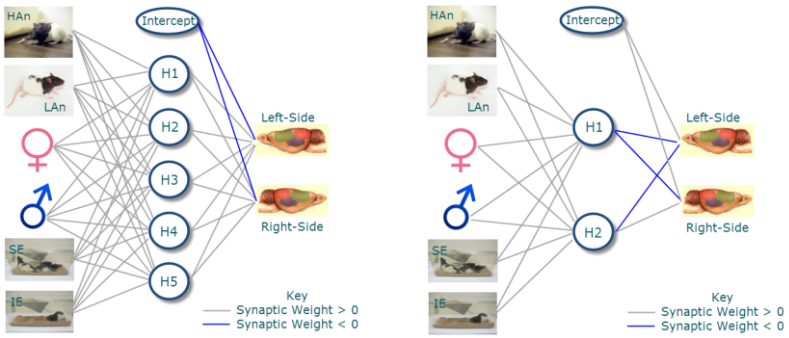


Figure 3. PAG (left) and Amy (right) MRANN Design

Table 3 provides the contribution of each input variable to the explanation of the variation in the target variables (*c-fos* counts). Interpreting the totality of results in table 3, sex dominates explanatory power in the PAG, Amy, Cg1, and Cg2 regions. Trait dominates in the NAc(sh) region while environment dominates in NAc(c) and VMH regions. The finding of elevated levels of *c-fos* in environment is supported by prior research of Martinez, et.al (2008). Their study reports how applying repeated shocks to animals in a stable environment resulted in increased dopamine levels in both the NAc(c) and NAc(sh) regions compared to when the animals were placed in a novel context. Environment also has a relatively high explanatory power in the PAG and Cg2 regions. Across all brain regions, Amy has the highest reported contribution for sex at 76.1% and the lowest reported contribution for environment at 6.5%. A finding that corroborates the findings of Amaral (2006) indicating that the amygdala is more reactive to sex than to environment.

Table 3. *MRANN Importance of Variables*

	PAG	Amy	Cg1	Cg2	NAc(c)	NAc(sh)	VMH
Sex	0.409	0.761	0.475	0.347	0.328	0.168	0.280
Trait	0.196	0.174	0.299	0.307	0.281	0.575	0.354
Environment	0.395	0.065	0.226	0.346	0.391	0.257	0.366
Total	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The rat model has delivered converging evidence to support the neuroscience translation to an optimizing public-housing assignment model. The innovative bridge across disciplines is exploited in the next section. Here we describe the neuroscience inspired MCDM optimizing assignment model for the allocation of subsidized apartments in the public-housing market.

MCDM and Optimal Prosocial Housing Assignment

The primary aim of this section is to translate the findings from the neurobehavioral rat study into a prosocial-aware public-housing apartment assignment model. We regularize the translation process by

enumerating equivalence between the terms of the animal model and the decision-making factors of the assignment model (Table 4).

Table 4. *Translational Equivalence*

Animal Model		Human Model
Subject: Rats	≡	Subject: Human
Trait	≡	Psychological Factors
• HAn		• Prone to Hypertension, Anxiety, Panic, Stress, etc.
• LAn		• More in control of Emotion
Sex	≡	Sex
• Male		• Male
• Female		• Female
Environment	≡	Housing / Building
• Isolated		• Impoverished
• Social		• Enriched
Behavior	≡	Migration
• EPM		• Entryways
• LMA		• Courtyards / playgrounds
Physiological		Physiological Markers
• BP and Temp.		• BP, Temperature, Pulse, etc. measured via ECG or PPG
Open field novel stress test /w AMPH	≡	An anxiety / stress inducing scenario to measure emotional response such as the Stroop Test, Stress Scale Questionnaire, Reaction Time Test, etc.
SIS Index	≡	SIS Index[9] – measure of antisocial spillover behavior: $0 \leq \delta \leq 1$

Next, as a branch of MCDM, we describe a combinatorial multiple objective assignment algorithm (MOAP) in the context of the translated components. The first component unifies the translation of the equilibrium prosocial housing model and the exploratory rodent experiment that resulted in a measurement methodology for per capita social indicator scores (SIS). The second component specifies the assignment of *n* objects (an eligible resident group, or family) to *m* other objects (an apartment) within the presence of multiple and hierarchical goal functions.

A Goal Programming Approach

The public-housing apartment assignment problem is characterized by a multiplicity of criteria for judging the optimal prosocial mix of eligible residents. The goal for housing managers is to attain more than one objective (goal) in selecting the portfolio mix of eligible residents for a specific housing space at a defined time. A feasible efficient solution to the decision-problem of assigning eligible residents to available apartments is achieved when the decision-maker is able to minimize deviations away from basic prosocial hierarchical objectives including the following: a) per capita antisocial interaction; b) total

[9] This is achieved by the administration of a social-survey questionnaire(s) and anxiety-inducing test(s) with the goal of extracting quantifiable physiological and anxiety markers to describe all eligible applicants (Apoorvagiri & Nagananda, 2015) in a factor structure that mimics the EFA latent dimensions uncovered in the rat experiment.

weighted travel time to facilities (i.e., the weighted time travel to the CBD); and, c) assigning "in-need" residents (anxious, substance abuse, etc.) to apartments with close proximity to the CBD. Prosocial goal constraints are presented in the next section. Mathematically, this problem can be represented as an *n* dimensional decision variable vector. Specifically, the problem consists of *n* decision variables, *m* constraints and *k* objectives where any or all of the functions may be nonlinear. The mixed-integer nonlinear goal programming algorithm (MINLGP) of Dash and Kajiji (2014) is a viable approach to solve this problem.

$$MINLGP = Min\ Z = \left[P_1(h^-, h^+),\ P_2(h^-, h^+),\ \ldots,\ P_L(h^-, h^+) \right]$$

$$S.T.\quad Ax + Bf + h^- - h^+ = b$$

$$x, f, h^-, h^+ \geq 0\ ,\ f \in \mathcal{Z}^F,\ \text{and,}\ x \in \Re^{l-F}$$

where *k*, the number of goal constraints is such that $A \in \Re^{kxl}$; $B \in \mathcal{Z}^{kxF}$, $b \in \Re^k$, and *Z* quantifies the attainment of *L* hierarchical levels such that $P_1(h^-, h^+) > P_2(h^-, h^+) > \ldots > P_L(h^-, h^+)$ when implicated, scaling effects are applied to $P_l(h^-, h^+)$. In the absence of integrality constraints, $F = 0$. The specification *b* is the *k*-component vector of goal targets and h^- and h^+ are *k*-component column vectors that capture goal under- and over-achievement, respectively. Lastly, we define the optimal solution to the convex MINLGP, x^*, as the one that satisfies all hierarchical goals as much as possible.

Understanding that the simple assignment problem (e.g., $m = n$) is a bijective mapping of a finite set $N = \{1, 2, \ldots, n\}$ into itself, we add and augment the well-known assignment constraints of (Burkard & Çela, 1999) to account for complexity goals (i.e., $m \neq n$). MOAP specification is obtained by incorporating the following goal constraints:

$$\sum_{i=1}^{n} x_{i,j} + h^- - h^+ = 1\ for\ all\ j = 1, 2, \ldots, m$$

$$\sum_{j=1}^{m} x_{i,j} + h^- - h^+ = 1\ for\ all\ i = 1, 2, \ldots, n$$

$$x_{i,j} \in \{0, 1\}\ \forall\ \text{i and j}$$

Maximum Coverage for Prosocial Apartment Migration

Extant research provides evidence that an efficient solution to the assignment location problem is one that hierarchically weights both social and economic objectives (for a discussion on constrained economic decisions, see: (Church & Velle, 1974; Glimcher, Dorris, & Bayer, 2005); and (H. K. Smith, Harper, & Potts, 2015)). Following the contributions of Bellomo et.al. (2017), the theoretical MOAP proposed in this research is not bound by the assumption of rational socio-economic behaviors. The multiobjective

and multicriteria extensions specifically capture the interplay among neuroscience, psychology and social systems (e.g., healthcare delivery, social well-being, etc.) while assuring maximal apartment coverage for eligible residents (Trotochaud, 2006).

Exploratory Prosocial Goal Constraints

The MRANN results of the rat experiment provide evidence that sex was significant in explaining the variation in *c-fos* levels of Amy. The behavioral and physiological assays of Liu et. al (2018) indicate that both sex and environment did modulate the stress and anxiety of the rats. Based on these findings as well as the extant findings, we proceed to develop the MOAP goal constraints for the exploratory model as stated in equations (15) to (24):

$$\sum_{i \in I} HAn\left(x_{i,OS}^{male}\right) + h^{-} - h^{+} = y_i \tag{15}$$

$$\sum_{i \in I} HAn\left(x_{i,IS}^{fem}\right) + h^{-} - h^{+} = y_i \tag{16}$$

$$\sum_{i \in I} LAn\left(x_{i,IS}^{male}\right) + h^{-} - h^{+} = y_i \tag{17}$$

$$\sum_{i \in I} LAn\left(x_{i,OS}^{fem}\right) + h^{-} - h^{+} = y_i \tag{18}$$

$$\sum_{i \in I} AMPH\left(x_{i,IS}^{male}\right) + h^{-} - h^{+} = y_i \tag{19}$$

$$\sum_{i \in I} AMPH\left(x_{i,IS}^{fem}\right) + h^{-} - h^{+} = y_i \tag{20}$$

$$\sum_{i \in I} y_i m_{i,IS} \leq m_{IS} \tag{21}$$

$$\sum_{i \in I} y_i m_{i,OS} \leq m_{OS} \tag{22}$$

$$\sum_{i \in I} y_i m_{i,IS} + \sum_{j \epsilon I} y_j m_{j,OS} \leq m \tag{23}$$

$$\sum_{i \in I} \sum_{j \epsilon m_i} SIS_{i,j} x_{i,j} - h^{+} = 0 \tag{24}$$

Where $x_i = (0,1)$ for all $j \in m$; $y_i = (0,1)$; m_i denotes available apartments for assignment in the *i-th* building. *I* denotes total eligible apartment buildings in the city bounds. In this scheme, buildings where travel times to the CBD are considered short are located *Inside-S (IS).* By contrast, buildings *Outside-S* (*OS*) are structures located at a distance beyond *S* where service delivery is diminished and travel times to the CBD are considered long. Additionally, we define the following two variables:

$$x_{i,IS} = \begin{cases} 1 & if\ a\ eligible\ resident\ is\ assigned\ to\ an\ IS\ building \\ 0 & otherwise \end{cases} \text{ and}$$

$$x_{i,OS} = \begin{cases} 1 & if\ a\ eligible\ resident\ is\ assigned\ to\ an\ OS\ building \\ 0 & otherwise \end{cases}.$$

Goal constraints (15) and (16) are active (i.e., $y_i = 1$) only when a high-anxiety male (female) head of household is assigned to an apartment *Outside S* (*Inside-S*). Similarly, goal constraints (17) and (18) are active only when a low-anxiety male (female) head of household is assigned to an apartment *Inside-S* (*Outside S*). Goal constraints (19) and (20) seek to make apartment assignments *Inside-S* for male or female residents with substance abuse issues. Constraints (21) and (22) bind total placements of *Inside-* and *Outside- S* while equation (23) places a limit on total assigned apartments. The prosocial objective stated in equation (24) is to minimize the cumulative community-wide social indicator index, $SIS_{i,j}$, in the assignment of resident *i* to apartment *j*.

The Exploratory Prosocial Hierarchical Objective Function

The lexicographic MOAP is tied together by the augmented prosocial hierarchical priority structure. The decision problem contains one or more goals with the possibility of multiple scaler-weighted sub-goals. For the purposes of demonstration, we state the following hierarchical goal function as one possible statement in equation 25:

$$Min\ Z = \left\{ P_1\left[h_{24}^{+}\right], P_2\left[h_{18}^{+}\right], P_3\left[h_{19}^{-} + 2h_{20}^{-}\right] \right\} \tag{25}$$

Although the proposed model statement was constructed to demonstrate a commitment to seek prosocial apartment assignments across regulated public-housing units by assigning over-achievement of equation (24) to priority one. We note, however, that in actual practice it is entirely possible deviation from other social and system goals may need to also be considered at the priority one level. The second priority level characterizes managerial policy initiatives. This priority seeks to minimize goal deviation in the assignment of low-anxiety females to apartments *Outside S*. The goal to assign eligible residents who are substance-dependent *Inside-S* is stated at the third priority level. Lastly, within the third priority level, there are two sub-goals which have differential cardinal weights. By specification, compared to males, it is twice as important to minimize under-achievement in the assignment of females to apartments *Inside-S*.

Discussion

We conclude with a discussion of the practical implementation of the MOAP in the context of the multi-building public-housing assignment problem. First, the research provided a housing density theory for control of antisocial effects by building and by housing community. The theoretical model provided a framework for efficient apartment assignment. Within this theory subsidized residential alternatives were

indistinguishable to the population of eligible residents. Hence, optimal prosociality assignment in apartment mix is available for statement as an independent goal that coexists with any broader set of public objectives.

Second, with a prosocial housing theory in place, the research turned to a neurobehavioral animal model to estimate a rodent-oriented test of the prosocial density hypothesis. The abstracted neurobehavioral study investigated the role of environment, trait, and sex on rat behavior. After establishing a framework for the efficacy of translating rat studies to human decision-making models, we proposed a translation scheme based on the creation of a normative weighted per capita SIS metric. Within the context of the rat experiment, the SIS metric provides an expression of how alternate rearing environments impact the differing modulation of stress levels by biological sex and propensity to spill antisocial sociocultural experiences. As a decision variable attribute, in theory, the proposed human oriented SIS metric would serve as the translational link between the neuroscience of prosocial behavior and the MCDM housing assignment model.

Third, we proposed a baseline MCDM in the form of a multiple objective assignment model. This foundational model did not address some obvious and needed extensions. Age considerations (the elderly) and residents with disabilities are just two obvious extensions that must be added to complete a formal normative test on the domain of applicability. This task is left for future research. One recommended approach for future research is to simulate alternate urban growth scenarios (Jeon, et.al., 2018). Specifically, the alternate scenarios would simulate various demographic portfolios of the resident population seeking assignment to public-housing. The future exploration of translational neuroscience and housing science offers a promising and novel approach to achieving efficient apartment assignment with a prosocial outcome in urban public-housing communities.

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