Dynamic Lot Sizing and Promotion Decision with the Impact of UK Tax Legislation

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Abstract

Operational Research provides the methods and techniques by which firms can maximise their profits by taking smart decisions. The OR literature in the area of logistics, however, pays scant attention to cash flows that arise in order for the firm to fulfil its legal obligations. This research develops a methodology for constructing models for inventory management that explicitly account for taxation schemes. It does this by expressing the future profits of the firm after tax as the Net Present Value or Annuity Stream Value of the cash flow function associated with the activity for the firm, including the cash flows exchanged with relevant third parties and the government that are needed in the context of ensuring compliance with tax legislation. Using the legislation in the United Kingdom, the research establishes how the explicit consideration of the schemes by which Value Added Tax (VAT) and Corporate Tax are settled with HMRC may affect the inventory and promotional decisions of the firm. In this paper we focus on the case of the dynamic lot sizing problem. A fairly robust insight is that optimal ordering decisions often become synchronised with the VAT/CT tax return points within a standard corporation tax scheme, while the benefit of considering taxes explicitly into the decision models increases the higher the product value and/or the lower the marginal profit on a product. We also investigate a variation of this problem in which the company can choose the timing and duration of sales promotions in the case of price-sensitive demand.

Keywords— Dynamic Lot Sizing,Promotion,Tax,Cash Flow

1 Introduction

Several types of taxes exist and each may have a different impact on demand but also on how producers or distributors decide on strategic, tactical, or operational decisions. The sugar tax levy introduced in 2018 in the UK, for example, aims to stimulate producers to make strategic decisions about the sugar content in soft drinks. An import tax, for example, may affect the portfolio imported into a domestic market. In this paper we wish to address two types of taxes that most firms and products are subjected to: Corporate Tax (CT) and Value Added Tax (VAT).

In particular, we wish to investigate how CT and VAT may affect a firm's profitability through operational decisions on lot-sizing (when and how much to produce or procure) and promotions (when to introduce a period of a reduced sales price). The context in which this is investigated is the well-known single-product dynamic lot-sizing model of Wagner and Whitin (1958). Demand over future time periods may change but in addition, can be increased by introducing a promotion window, a number of periods in which the sales price is reduced. Rather than minimising total production costs (inventory and acquisition costs), the firm is now interested in maximsing total profits over a planning horizon.

In the UK, as in most EU countries, consumption tax is collected via a VAT system such that each firm collects VAT on sales and pays VAT on purchases, whereby the net difference is to be settled with the government at different times, typically much later than the moments when the VAT cash-flows exchanged with customers and suppliers initially arose. A similar delay occurs for the settlement of CT. This research looks explicitly at the flows of these payments and their timing, using an approach as presented in Beullens and Janssens (2014), as to uncover their potential impact on operational decision making. The timing of tax payments is specified by government rules, however the timing of operational and sales cash flows can be adjusted by the firm. We thus want to investigate whether production and promotion decisions would be affected by the cash flow of tax within a planning horizon, as to arrive at insight about when firms would benefit most from adopting a tax-adjusted model.

For the sake of making a quantitative analysis, we assume the following setting (which will be further adjusted when necessary). A firm receives payments from customers by selling at unit price $p(1+\tau)$, where p is the sales price and τ the VAT rate. If y products are sold annually, the firm will have to pay the OUTPUT VAT $py\tau$ to the government, typically in several instalments at certain times in the future which correspond to so-called VAT tax points. The firm incurs acquisition costs at a price $c(1+\tau)$ per product. Per year, the firm can reclaim the INPUT VAT amount $cy(1+\tau)$, typically again in installments at the VAT tax points. Assuming the firm operates a profitable business, the Corporation Tax (CT) payment would be based on the annual operational profit $(p-c)y(1-\epsilon)$, where ϵ is the CT rate. If we don't account for the timing of these (and other) cash-flows involved in this activity, then both the VAT and CT rates do not need to be incorporated in a model about deriving optimal operational decisions.

To take account of the timing of tax-related cash-flows, we construct a model that aims to determine which operational decisions will maximise the Net Present Value (NPV) of this activity for the firm. The NPV is in general the Laplace transform of all relevant cash-flows involved in this activity for the firm, where the Laplace frequency represents the opportunity cost of capital rate α for the firm, see e.g. Grubbström and Kingsman 2004. By solving the model for $\alpha = 0$, we would get a solution from a 'classic' model that does not account for VAT and CT. We can deduce the value of using the tax-adjusted model by calculating the gap between the profitability of the classic model's optimal decisions versus the optimal decisions found from the tax-adjusted model. In calculating this profitability gap, we obviously need to use the NPV objective function which does account for the tax cash-flows.

Extensive numerical experimentation with our modelling framework leads to the fairly general result that in particular the VAT tax points affect the timing and lot-sizing of production for the firm. If the firm considers promotions, then the VAT tax points may also affect when it is best to offer these promotions, all other things remaining constant. As output cash-flows (from sales) are larger in magnitude than input cash-flows (from product acquisition), while the same VAT rate applies to both, promotional planning may be more important that acquisition planning, although the best operational results are obtained by considering both options, possibly leading to an increase of a few percentages in NPV compared to a classic dynamic lot-sizing/promotion model. In contrast to the current literature which so far has, to the best of our knowledge, ignored the impact from VAT on operations, we thus find that VAT may have in certain circumstances an influence on optimal operational decisions.

We refer the reader to net present value of dynamic lot sizing problem see Grubbström and Kingsman (ibid.), Lim et al. (2013) and Bian et al. (2018) for related work on dynamic lotsizing with a net present value criterion; to Thomas (1970) and Sogomonian and Tang (1993) on earlier work on dynamic pricing and promotion strategies, and to Wu et al. (2017) and Brahimi et al. (2017) for pricing decisions in dynamic lot-sizing models.

2 Dynamic Lot Sizing with Tax Implication

Notation

N	dynamic lot sizing planning horizon
n	number of setups to be performed, between 1...N
π	profit function
R_i	tax adjusted revenue in period i
C_i	tax adjusted expenses in period i
H_i	tax adjusted holding cost in period i
$R(i)$	tax adjusted revenue from period $i+1$ to N
$C(i)$	tax adjusted expenses from period $i+1$ to N
$H(i)$	tax adjusted holding cost from period $i+1$ to N
T_a	yearly basis
$CT(i)$	corporation tax return day on ith period
$VT(i)$	value added tax return day on ith period
p_i	unit selling price in period i
y_i	demand in period i
c_i	unit purchasing price in period i
s	fixed set up cost
α	opportunity cost of capital
Q_i	production quantity in period i (Decision Variable)

The objective of this tax-adjusted model is to maximize the NPV of the profit (revenues minus costs) after tax. The revenue is a function of units sales price and demand. The costs are expenses related to unit acquisition and holding products in stock. In addition, all cash-flows related to VAT and CT are also to be incorporated.

- NPV of After Tax Revenue in Period i : receive payment for demand of y_i with output VAT payment from the customer denote as $p_i y_i e^{-\alpha(i-1)}(1+\tau)\frac{(1-e^{-\frac{\alpha}{T_a}})}{\alpha}$, $e^{-\alpha(i-1)}$ is time delay factor, $\frac{(1-e^{-\frac{\alpha}{T_a}})}{\alpha}$ denote sells happens on daily basis ; this output VAT $p_i y_i \tau$ should be paid back to the government based on the VAT scheme, it can be in day 30,60,90 and we denote this VAT return day by $VT(i)$, hence, the output

VAT payment day written in time delay factor as $-p_i y_i \tau e^{-\alpha \frac{VT(i)-(i-1)}{T_a}}$; for the revenue of $(p_i - c_i)y_i$, need to pay corporation tax of τ base on corporation tax scheme, the due date can be 30,60,90 and we denote this corporation tax payment day by $CT(i)$, hence, corporation tax payment for this transaction is $-(p_i - c_i)y_i \epsilon e^{-\alpha \frac{CT(i)-(i-1)}{T_a}}$. Add all these three terms displays as R_i.

- NPV of After Tax Expenses in Period i : operational related cost is mainly composed with set up cost and purchasing cost according to dynamic lot sizing problem, VAT payment for these cost with time delay factor note as $[s(1+\tau) + c(1+\tau)y_i]e^{-\alpha(i-1)}$; VAT payment for this cost can be reclaim back on VAT return day denote as $VT(i)$, hence, VAT reclaim back for both set up and purchasing cost is $-(s+cy_i)\tau e^{-\alpha \frac{VT(i)-(i-1)}{T_a}}$; for the setup cost it can be deduct from corporation tax payment on corporation tax return day and can be written as $-s\epsilon e^{-\alpha \frac{CT(i)-(i-1)}{T_a}}$. Combine these operational expenses can be displayed as C_i.
- NPV of After Tax holding cost in period i. it is compose of basic holding cost term $fI_i e^{-\alpha i}$, consumption tax of VAT payment for this holding cost $-f\tau I_i e^{-\alpha \frac{VT(i)-(i-1)}{T_a}}$ and corporation tax deduction from the profit on time i for the amount of $-f\epsilon I_i e^{-\alpha \frac{CT(i)-(i-1)}{T_a}}$. Holding cost term in period i add all these terms denote as H_i.

The total objective function express as below:

Problem 1

$$\begin{array}{lll} \max & \sum_{i=1}^{N} [R_i - E_i - H_i] & \\ \textit{subject to} & I_i = I_{i-1} + Q_i - y_i & \forall i = 1, ..., n \\ & Q_i \le Mx_i & \forall i = 1, ..., n \\ & x_i \in \{0,1\} & \forall i = 1, ..., n \\ & I_i, Q_i \ge 0 & \forall i = 1, ..., n \end{array}$$

Since it is possible (see also the equations presented further below) to condense the NPV of all cash-flows, including the tax-related cash-flows, into the parameter format of the classic dynamic lot-sizing formulation, we can deploy any solution method to the classic problem to solve our tax-adjusted model. Our approach follows broadly the method of Gilbert (1999). The problem was solved as a cost minimization allowing solutions with exactly n setups, where n is a number then varied between 1 and N (the planning horizon). They identified this minimizing value involves only determination of the best n times of setup. As the setup is fixed it is only concerning to minimizing the cost of holding inventory. In our problem, as we consider time value of money invested in the inventory, we do need to consider the total profit function rather than only holding costs. Note that the Wagner and Whitin (1958) theorem still holds. As there is zero inventory but non-zero demand in first period, a setup is required in the first time period.

$R(i)$ is the after tax revenue from satisfying demand from periods $i+1$ to N:

$$R(i) = \sum_{k=i+1}^{N} \Big[p_k y_k e^{-\alpha(k-i-1)}(1+\tau)\frac{(1-e^{-\frac{\alpha}{T_a}})}{\alpha} - p_k y_k \tau e^{-\alpha \frac{VT(k)-i}{T_a}} - (p_k - c_k)y_k \epsilon e^{-\alpha \frac{CT(k)-i}{T_a}} \tag{1}$$

$C(i)$ are the total of setup and acquisition costs covering periods $i+1$ to N:

$$C(i) = s(1+\tau)e^{-i\alpha} + c(1+\tau)\sum_{k=i+1}^{N} y_k e^{-i\alpha} - (s + c\sum_{k=i+1}^{N} y_k)\tau e^{-\alpha \frac{VT(i+1)-i}{T_a}} - s\epsilon e^{-\alpha \frac{CT(i+1)-i}{T_a}} + H(i) \tag{2}$$

$H(i)$ is the holding cost covering periods $i+1$ to N:

$$H(i) = f\sum_{l=2}^{N-i}(y_{l+i}\sum_{k=1}^{l-1} e^{-k\alpha T}) - f\tau \sum_{l=2}^{N-i}(y_{l+i}\sum_{k=i+1}^{l+i-1} e^{-\alpha \frac{VT(k)-i}{T_a}}) - f\epsilon \sum_{l=2}^{N-i}(y_{l+i}\sum_{k=i+1}^{l+i-1} e^{-\alpha \frac{CT(k)-i}{T_a}}) \tag{3}$$

The solution method of Problem 1 tax considered profit maximized problem is re-formulated below. Find maximum profit value within the n times set up displayed in function 4, and in order to find this value need to find maximum i value when set up n is fixed use function 5, further achieve this result, use recursion method in function 6 and function 7.

$$\pi = \max_{n \in (1,...N)} \{\pi_n\} \tag{4}$$

$$\pi_n = \max_{i \in (0,...N-1)} \{g_n(i)\} \tag{5}$$

$$\underset{t>i, t\le(N-(n-1))}{g_n(i)} = \max\{\sum_{k=i}^{t-1}(R_n(i) - C_n(i)) + g_{n-1}(t)e^{-\alpha \frac{t-i}{T_a}}\} \tag{6}$$

for n=2,...., N and i=0,....,N-n.

$$g_1(i) = \sum_{k=i}^{N-1} (R_1(i) - C_1(i)) \tag{7}$$

for i=0...,N-1.

3 Promotion strategy in Dynamic Lot Sizing with Tax Implication

For the dynamic pricing, Thomas (1970) considered a demand function and cost parameters that may vary over the time and a forward dynamic programming algorithm was adopted. Profit from node i to j is given by:

$$\pi_{ij}(p_{ij}) = \sum_{k=i}^{j-1} (p_{k+1} - c_{i+1} - (k-i)h)d_{k+1}(p_{k+1}) - K_i \tag{8}$$

For $0 \leqslant i \leqslant j \leqslant N$ which means node i to j, and the price vector $p_{ij} = [p_i, ..., p_j]$,π_{ij} is the total profit if the production takes place in node i to satisfy demands in periods $i+1, i+2, ..., j$. Define subplan which consisting of periods $i, ..., j$.

$$\pi_{ij} = \max_{p_{ij}} \{\pi_{ij}(p_{ij})\} \tag{9}$$

In this method, Thomas (1970) shows that if a setup takes place in period i and the next setup in period j, then the optimal price for period $k = i, ..., j-1$ must be set at the value which maximizes profit.

We apply the same forward dynamic programming method to decide the promotion strategies and ordering quantities in our problem. The original selling price can be denoted as po and related demand can be $y(po)$, and the promotion price use pp and relatives demand function is $y(pp)$. The profit function display as below with the difference of revenue $(R(ij))$ and cost $(C(ij), H(ij))$, it follows the equation of 1,2 and 3 method:

$$\pi_{ij}(p_{ij}) = R(ij) - C(ij) - H(ij) \tag{10}$$

The objective is to find the price that maximises the value of profit in specific time period as in (11).

$$\pi_{ij} = \max_{p_{ij} \in (po, pp)} \{\pi_{ij}(p_{ij})\} \tag{11}$$

$$F(t) = \max_{i=1,...,t} \{F(i-1) + \pi^*_{ij}\} \tag{12}$$

The label $F(t)$ represents the longest path or maximum profit path until period t. In a forward pass, we start labelling node 0, node 1, ..., up to node n. Then $F(n)$ provides us with the total maximum profit solution.

4 Numerical experiments: some illustrative results

Table 1: Experiment with classical versus tax adjusted DLSP

$p = 12$	$c = 10$	$s = 10$	$\pi^t = 457$	$\pi = 454$						
Tax	TPeriods	0	60	90	150	180	240	270	330	365
	TQuantity	60	30	60	30	60	30	60	30	
Classical	Periods	0	60	120	180	240	300	365		
	Quantity	60	60	60	60	60	67			

Table 2: Promotion strategy with classical versus tax adjusted DLSP

c=10	s=11	36%	$po = 12$	$pp = 11.5$	$\pi_t = 902.69$	$\pi = 876.37$	3%		
Tax	TPeriods	0	49	90	139	180	229	270	321
	TPrice	2	1	2	1	2	1	2	1
	TQuantity	127	78	127	78	127	78	133	84
Classical	Periods	0	46	92	138	184	230	275	320
	Price	1	1	1	1	1	1	1	1
	Quantity	88	88	88	88	88	86	86	86

This is numerical test shows how the inventory ordering policy changes with the tax considered scenario. First in Table 1 example with $p = 12, c = 10$, without tax consideration there is same ordering quantities obtained

(60,60,60...), while, the tax added algorithm shows more order place before tax return day (both VAT and CT return day is 90,180,270,360) and follows with one more small orders. The main reason is comparable low setup cost, just place large amount of order before next tax return, then all the input tax can be claimed back and can hold output tax from the selling for more longer time. This result shows how operations can take advantage of input tax payment in inventory decision.

Further look at Table 2 for promotion strategy ($c = 10, s = 11, y = 700$), see how the output tax payment can affect the operational decision. The orginal price of 12 goes with demand of 700, but if it promotion price of 11.5, demand will increase by 36% from original demand. For the tax mode the promotion price decided with the same pattern of the tax due payment time while the classical method give the general result of original price (original price 1, promotion price 2). This difference promotion price decision can end up of 3% profitability difference. The same reason as $py(1+\tau)e^{-\alpha i}$ is the output VAT for the selling, and pay back in some point $py\tau e^{-\alpha VT(i)}$, the longer time keep output VAT , the better for the cash flow for the operation, this is the main reason do promotion on VAT and CT cycles, which can boost demand and also purchasing happens before VAT return day or on VAT return day can be reclaimed back immediately for the input tax. Therefore, even the same profit margin, with higher unit selling price products should take much more advantage of these features of tax payment.

From experiment can find out operations should fully take advantage of the holding output tax longer and input tax shorter, especially in both selling and purchasing price high scenario, the comparable high selling price even the same low profit, as the base price is higher, both in and out flow of tax has much more impact compare to the lower price scenario; High set up cost case, the inventory ordering pattern comply with tax return policy but if too large it escape one or two periods come to next tax return time, the reason is the operational cost relatively high, revenue generated between the tax return cannot cover for the comparable high set up cost even account for the input tax cash flow payment; Demand just comes after the tax return time, inventory ordering before tax point to reclaim input tax, at the same time the out tax can be kept to next tax point. Using Classic model, the synchronization with demand occurs but operations will order too late so that the input tax has to be reclaimed only at next tax point.

With Tax considered promotion strategy inevitably do promotion just before the tax return time which induce more demand and boost the selling. Input tax (payment from the customers) gives the extra benefit of the cash, however, in some point any cost change go over this benefit of tax inflow, it is not worth to do promotion price. From these we summarize following things. First need to find out what is the boundaries of these change, this means in which case the tax model and traditional model have different price decision. It is depend on the selling price and cost further for the setup cost and demand pattern. Second it is important, decide the scope of the promotion plan change in tax adjusted model because consideration of tax gives the extra benefit of the cash, as long as cost change does not go over this benefit of cash in flow (input tax), tax model keep it promotion price strategy. Third, tax point is important. Except tax adjusted model choose all promotion price, when it choose mixed plan of two different prices, the promotion price always decides on the tax payment periods comes with larger ordering quantities.

5 Conclusion

Adopt the tax payment structure in classical dynamic lot sizing problem and precisely considered in and out flow of tax in the operational process. First, we propose a new model allows us to measure how the tax flows has impact on the timing and ordering decision. Secondly, based on this result, further investigate promotion strategy with dynamic lot sizing problem consider both tax and time value of money. In tax adjusted dynamic lot sizing problem, ordering time and quantities changed based on the government tax due date and which gives higher profitability compare the literature model. Hence, it is benefit for the operations take advantage of tax payment structure in operational strategies. The benefit of this method is any delay or advance payment for the tax purpose in the supply chain should treat as cost or extra cash flow in in inventory related operational decision.

Deriving profit functions from the NPV framework instead of the traditional average profit method has several advantages. The largest advantage of the NPV is accounting the in and outgoing cash flows do not have to coincide with the physical transactions of the products move through the system and this advantage comes along with the features of in and out flow of tax payment. This approach is more accurate in that it can easily model the systems where operational activity happens from the moment and actual tax payment happens in some point in future(output VAT and CT), and considered further payback of the input tax. To the best of the authors' knowledge, tax with production and inventory systems always modelled in average cost function, this study especially contribute the process of tax collection for the government has impact on the logistic decisions and related activities. This study introduces the concept of the tax point and shows how NPV profit function can easily construct the cash flows of actual tax in supply chain.

For future research consider multiproduct dynamic lot sizing problem combined with supplier in different locations to find out whether tax flows has impact on the supplier selections.

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