A.M.Lyapunov theory methods in complex multi-scale systems dynamics

Lyudmila K.Kuzmina
Kazan Aviation Institute (Kazan State University)
RUSSIA
Lyudmila.Kuzmina@kpfu.ru

True theory cannot be linear **A.Einstein**

It is important study, that is concerned with the development of the concepts and methods of classical stability theory in reference to the problems of singularly perturbed class systems. The various aspects of complex multi-scale systems dynamics are considered. Methods of the modelling and analysis on the generalized methodology basis, combining the Lyapunov stability theory ideas and asymptotic theory manners, are elaborated. Non-traditional, extended approach, formed on rigorous mathematical Lyapunov's methods, Chetayev's stability postulate and the singularity postulate, is worked out. It gives universal mathematical tool, that makes it possible to come near to the solving of fundamental problems in general qualitative analysis for singular systems dynamics, including reduction/decomposition problems of complex systems.

Here generalized statements and stability problems for singularly perturbed class systems are considered. The matter of investigation is the objects, for which original mathematical models are presented in standard form of singularly perturbed systems with parametric perturbations of non-regular type. The critical transcendental cases (in A.M.Lyapunov sense), that are generated by applied problems of Mechanics, are examined. Besides the systems with peculiarities inherent for mechanical systems are considered: there is not the uniform asymptotic stability property; the perturbed system is close to boundary of stability domain; generating systems are non-limit, also singular ones; the nominal systems are quasi-Tikhonov's ones (on N.N.Moiseev). The reduction conditions are determined, under that the qualitative problems for original system is reduced to the investigation of shortened, approximate, system of less order; moreover – in general case this shortened system is singular one (in sense of A.N.Tikhonov, S.Campbell). Here it is obtained the solution of singularly perturbed problem of stability for cases, when the spectrums of corresponding matrices are critical (with zero real parts of eigenvalues both for slow and for fast variables). Regular algorithms for the estimating of parameters values, permitting the reduction in stability problem, are constructed. The results are discussed, that give the strong substantiation of reduction principle for considered class of multi-scale systems dynamics. The received results are new ones that are generalizing and supplementing well-known results, that are known in stability theory and in perturbations theory; and these new results are interesting both for theory and for engineering applications.

In the applications to Mechanics the elaborated methods are very effective, enabling to construct the acceptable shortened models (as comparison models in sense of R.Bellman, V.M. Matrosov) by rigorous mathematical way; to substantiate strongly their correctness in dynamics, including Lyapunov's critical cases; to consider specific cases, inherent in mechanical systems; to evaluate the corresponding errors in such transition-reduction. In framework of this approach it is considered actual problems (as examples) from Mechanics.

New elegant outcomes are obtained, that are interesting both for theory and for applications, both in general theory of singularly perturbed systems and in applied engineering problems; also this approach is very perspective from gnosiological view point, for general Knowledge theory.

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