# Nonlinear Hierarchical Modeling for Efficient Asset-Liability Management of Property-Liability Insurers [*]

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## ABSTRACT

Financial optimization is an important part of efficient decision-making under uncertainty. In the context of asset and liability management (ALM), this paper presents an optimization model for property-liability insurers that assures a simultaneous mean-variance (MV) allocation of asset- and liability-side resources when management is faced with a complex hierarchical goal environment. The optimization model presented in this paper relies upon a separable nonlinear goal-programming (NLGP) paradigm as a means by which to resolve the hierarchical and potentially conflicting objectives that govern efficient portfolio solutions in the face of governance constraints. Decision-making implications from solving the NLGP are explored by modeling the portfolio of a hypothetical property-liability insurer. The NLGP model extends previous optimization results by providing a nonlinear program that is capable of resolving multi-criteria ALM models that are characterized by MV efficiency decisions on both sides of the accounting identity.

# 1. Introduction

The liquidity strains of the late 1970s and early 1980s caused property-liability insurers to follow the example set by the U.S. banking industry in its development and use of asset-liability management (ALM) models as means by which to mitigate the ill effects of inefficient resource utilization caused by an increasingly complex economic environment. By the late 1980s banks had already advanced the state of ALM frontier methods in their quest for efficient economic asset strategies given a particular liability profile and business plan. For example, Booth and Bessler (1989) provide a mature specification of the bank ALM model to include futures-based hedging under uncertainty. Boender, et. al. (1998) provide a summary of contemporary ALM models and identify the stylized relationships needed to produce consistent decision-making scenarios. Subsequently, Korhonen (2001) generalized modeling efficiency within the context of multi-criteria decision-making. A robust example of the multi-criteria ALM applied to a Greek bank is provided by Kosmidou and Zopounidis (2002). More recently, Dash and Kajiji (2002) advance the synthesis of bank ALM structures by treating uncertainty, nonlinear demand and supply economies within a complex hierarchical decision-making environment.

It has long has been recognized that property-liability insurance firms face ALM complexities that are not dissimilar to those of the banking industry. Property-liability firms conduct at least two distinct activities. One is the selling of future claims against themselves or the issuing of insurance contracts. Investment diversification is the other activity. This activity accrues directly from the underwriting function. The industrial organization issues that surfaced with continued extensions to bank ALM models are now in evidence for property-liability

insurers (for a recent review of the empirical issues on consolidations and economies of scope, see Cummins et. al. (2003)). From the early ALM formulations of Hofflander and Drandell (1969), Drandell (1977), and Cummins and Nye (1981) property-liability insurers have, like banks, sought to improve ALM specificity. During the 1980s and 1990s the focus of ALM models was directed at developing a mesh between evolving financial theory and the efficient management of firm-specific risk.[1] Today, global capital market structures along with emerging new technologies have caused insurers into alliances on a scale that was unimaginable only a few short years ago. The Financial Institutions Modernization Act of 1999 (FIMA) abolished the historical separation between commercial banking and insurance underwriting. Implicitly, FIMA also extinguished the barrier between bank and property-liability ALM structures as well.

This paper proposes a nonlinear goal program to solve the ALM model of a typical property and liability insurer. The model building process has two focal points. First, there is the characterization of a separable programming-based multiple goal optimization method. Second, the focus shifts to an application that demonstrates how the algorithmic method is suitable for the simultaneous optimization of multiple mean-variance portfolio problems that may exist at different levels within the managerial goal hierarchy. Specifically, the paper

[1] See Lamm-Tennant, J. (1993). "Stock Versus Mutual Ownership Structures: The Risk Implications." Journal of Business **66**(March): 29-46. for a discussion on organizational form and the management of firm-specific risk. Ran BarNiv and James B. McDonald BarNiv, R. and J. B. McDonald (1992). "Identifying Financial Distress in the Insurance Industry: A Synthesis of Methodological and Empirical Issues." Journal of Risk and Insurance **49**(4): 543-574. provide a focused review of specific ALM methodologies that targets insolvency problems within the insurance industry. Similarly, Petroni, K. R. (1992). "Optimistic Reporting in the Property-Casualty Insurance Industry." Journal of Accounting and Economics **15**(4): 485-508. presents evidence that supports the adoption of ALM approaches by documenting how weak insurers are more prone to forego the use of ALM methodologies as way to avoid the inherent objectivity the method brings to the description of adverse news (eg., required estimates of claim losses). Cummins, D. J., M. A. Weiss, et al. (1999). "Organizational Form and Efficiency: An Analysis of Stock and Mutual Property-Liability Insurers." Management Science **45**: 1254-1269. focus on organizational form and efficiency for both stock and mutual property-liability insurers.

proceeds by developing the separable programming nonlinear goal programming method in section 2. Section 3 presents the '*generalized*' property-liability firm and a representative normative model specification. Because the purpose of this paper is illustrative, the model presented in this section does not focus on a specific firm but is purposely generalized to meet the needs of a typical major property-liability insurer. Following the presentation of the model, section 4 presents the data relating to the hypothetical property-liability insurer. Section 5 reviews the results obtained from solving the model. A summary and relevant conclusions close the paper in section 6.

## 2. Separable Programming for Nonlinear ALM Models

Babbel and Hogan (1992) provide a review on how an organizational structure and managerial incentives under goal conflicts impacts the prioritization of the ALM solution process. It is well established that goal programming is an appropriate and recognized optimization method by which to solve a decision-model that is characterized by a defined organizational structure, heterogeneous decision-making attributes, and hierarchical objectives. The nonlinear goal program applied to the property-liability ALM presented in this paper is summarized as follows:

$$Min \quad \sum_{p=1}^{k} Z_p \left[ \delta^+ h^+ + \delta^- h^- \right]$$

$$S.T. \quad Ax + Ih^- - Ih^+ = b$$

$$x, h^-, h^+ \geq 0$$

where $Z$ quantifies the $k$ objectives (goals) so that an optimal solution to the linear program associated with priority-one objectives is achieved prior to determining the optimal solution to

the linear program that defines priority-two objectives and so on. That is, in managerial governance of the generalized property-liability insurer it is assumed that resource allocations must be achieved in a prioritized sequence where $Z_1 > Z_2 > \ldots > Z_k$. $\delta^+$ and $\delta^-$ are constant terms that indicate relative preference within each *p-th* goal; A is an *m* x *n* matrix of technological coefficients created from the separable programming grid approximation; *b* is an m-component vector of goal targets; $h^+$ and $h^-$ are *m*-component column vectors that capture goal over- and under-achievement, respectively. Lastly, *x* is a *n*-component column vector of decision variables.

The method proceeds by solving a sequence of modified linear programs (*MLP*) that are consistent with separable programming constraints. The use of finite bounds on the decision variables permits the development of a piecewise linear approximation for each nonlinear function in the problem statement. To form a piecewise linear approximation using *r* line segments, it is necessary to select $r+1$ values of the scalar *x* over its range of $0 \le x \le u$. By the use of a grid of $r_j + 1$ points for each variable $x_j$ over its range, the separable programming problem in *x* becomes the following modified linear program in $\alpha$ where an adjacency criterion is imposed on the new decision variable $\alpha_{kj}$ by use of a restricted basis entry rule:

$$Min: f(\alpha) = \sum_{j=1}^{n} \sum_{k=0}^{r_j} \alpha_{kj} f_{kj}(\bar{x}_{kj})$$

Subject to:

$$g_i(\alpha) = \sum_{j=1}^{n} \sum_{k=0}^{r_j} \alpha_{kj} g_{kij}(\bar{x}_{kj}) \le b_i, i = 1..., m$$

$$\sum_{k=0}^{r_j} \alpha_{kj} = 1,\ j = 1,...,n \qquad \alpha_{kj} \geq 0, k = 0,...,r_j,\ j = 1,...,n.$$ [2]

Within the goal program we define $C_p$ as the cost vector for goals at the *p-th* priority level. The goal program proceeds to solve for the lexicographically smallest vector ($c_1x_1$, $c_2x_2$, …, $c_nx_n$). Given $\Gamma_1 = \{x \mid Ax = b, b \geq 0\}$ the goal program proceeds by solving the first linear program $LP_1 \equiv Min\{\dot{C}_1 x \mid x \in \Gamma_1\}$ with optimal solution at $x^*$. Then for each *k-1* remaining $x^*$, the goal program iterates through all specified priority levels solving an LP at each step: $LP_p \equiv Min[C_p x \mid x \in \Gamma_p]$, which implies that $x \in \Gamma_p = \{x \in \Gamma_{p-1} \mid \dot{C}_{p-1}x = \dot{C}_{p-1}x^*, j = 1,2,..., p-1\}$. For parsimony, we note that alternative methods exist by which to formalize multi-criteria decision-making in the treatment of resource allocation problems. See Novikova and Pospelova (2002) for an application to multicommodity networks.

## 3. The NLGP ALM Model for Property-Liability Insurers

The model presented here is forged from the generalized property-liability firm financial statement as presented in table 1. The model is a one-period deterministic characterization of the decision-making scenario. The notation is as follows. Beginning period value for asset is indicated by $A_n'$, *n = 1..N*. Similarly, the beginning period value for *m*-th liability account is

[2] Several researchers discuss the intricacies of separable methods. See Stefanov, S. M. (2001). Applied Optimization: Separable Programming, Dordrecht: Kluwer. **53:** 336 pages. for both theory and methods. For additional discussion, also see Drandell, M. (1977). "A Resource Association Model for Insurance Management Utilizing Goal Programming." The Journal of Risk and Insurance **44**(June): 331-315., Feijoo, B. and R. R. Meyer (1988). "Piecewise-Linear Approximation Methods for Nonseparable Convex Optimization." Management Science **34**(3): 411-419., and Fare, R., S. Grosskopf, et al. (1988). "On Piecewise Reference Technologies." Management Science **38**(12): 1507-1511..

indicated by $L_m'$ for *m =1..M*. A buy or sell decision on *n-th* asset is indicated by $B_n$ and $S_n$, respectively. The rate used to make the buy and sell decision on *n-th* asset is captured by $r_n$.

<<< Insert Table 1>>>

Insurance lines are defined in table 2. The model specification includes eight insurance lines that are divided into property lines (**P** lines) and non-property lines (**non-P** lines). The distinction is necessary to capture the traditional regulatory structure. The premiums collected on all insurance lines is captured by $P_p$, for *p = 1..P* lines.

<<< Insert Table 2>>>

### 3.1 ***Parameters***

The parameters needed to complete the specification of the model are listed in table 3. These parameters include both endogenously and exogenously determined characteristics of asset-liability behavior.

<<< Insert Table 3>>>

### 3.2 ***NAIC***

The National Association of Insurance Commissioners (NAIC) implemented the Insurance Regulatory Information System (IRIS) to monitor the financial condition of property-liability firms. The IRIS system is defined by eleven ratios categorized into four financial domains. The four domains are: a) overall ratios, b) profitability ratios, c) liquidity ratios and d) reserve ratios. The Commissioners also identified ranges that are considered usual for a healthy

insurer. Where appropriate, the generalized model developed below references a specific IRIS ratio or a risk domain of the IRIS system.

### 3.3 ***The Hierarchical Objective Function***

In the goal programming formulation the objective function ties together the equations used to express the various economic and accounting linkages. Because of its importance in tying together the NLGP model, the specification of the objective function (equation 0) is presented after we develop the model.

### 3.4 ***Asset-Liability Efficiency Decisions***

The property-liability company's firm manager faces two efficient diversification problems. One is the efficient asset allocation of *N* investment portfolio securities. The other is found on the liability side. It is the efficient balancing of operational risks incurred by the underwriting function across *M* insurance lines. Equations (1) through (10) define the two portfolio problems. Equations (1) through (5) specify the asset allocation process for investment securities, while equations (6) through (10) define the diversification of operating risk across underwriting lines. Portfolio optimization is formulated following the specification of the quadratic Sharpe (1963) diagonal model (see Krokhmal, et al. (2001) for recent extensions that include value-at-risk). Inputs to the diagonal optimization model are obtained by application of the single-index market model to the individual assets: $R_{S_i} = a_i + \beta_i R_M + \varepsilon_i$, where $a_i$ is the component of security *i*'s return that is independent of the stock market's performance, $R_M$ is the rate of return on the market index, and $\beta_i$ is a constant that measures the expected

change in $R_{S_i}$ given by a change in $R_M$, and $\varepsilon_i$ is a random element with an expected value of zero and variance $\sigma^2_{\varepsilon_i}$. that is used a measure of unique (unsystematic) risk of the *i-th* security. In addition to $\sigma^2_{\varepsilon_i}$, which is the measure of unique (unsystematic) risk for the *i-th* security, the Sharpe diagonal model also requires the variance of the market index, $\sigma^2_{R_M}$, which is specified as the *N+1* asset.[3]

### 3.4.1 *Optimal Investment Account Management*

Equation (1) and (2) measure unsystematic and systematic risk, respectively for *N* investment securities. In (1) the *N+1* security is the market index and $\sigma^2_{S_{N+1}}$ is its variability. The systematic risk level of individual asset relative to the market index $\beta_{S_p}$ is captured in (2). Equation (3) forces the portfolio to be fully invested (short-sales are not permitted). Equation (4) is the goal constraint for the managerially determined portfolio return, $R_{S_p}$. Equation (5) is an accounting restatement of return.

$$\sum_{i=1}^{N+1} \sigma^2_{S_i} x_i - d_1^+ = 0 \qquad (1)$$

$$\sum_{i=1}^{N} \beta_{S_i} x_i = \beta_{S_p} \qquad (2)$$

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[3] The method employed here traces back to Cohen, K. J. and J. A. Pogue (1967). "An Empirical Evaluation of Alternative Selection Models." Journal of Business **40**(2): 166-193. Representative extensions can be found in Frankfurter, G. M. (1976). "The Effect of Market Indexes on the Ex-post Performance of the Sharpe Portfolio Selection Model." The Journal of Finance **31**: 949-955. For a summary of alternative computational methods, see Elton, E. J. and M. J. Gruber (1997). "Modern Portfolio Theory, 1950 to Date." Journal of Banking and Finance **1997**: 1743-1759. More contemporary extensions that demonstrate alternative approaches is expemplified by Jobst, N. J., M. D. Horniman, et al. (2001). "Computational Aspects of Alternative Portfolio Selection Models in the Presence of Discrete Asset Choice Constraints." Quantitative Finace **1**: 1-13..

$$\sum_{i=1}^{N} x_i = 1 \tag{3}$$

$$\sum_{i=1}^{N} r_{S_i} x_i + d_4^- - d_4^+ = R_{S_p} \tag{4}$$

$$C_{r_s} = \sum_{i=1}^{N} x_i r_{S_i} \tag{5}$$

3.4.2 *Optimal Insurance Line Diversification*

The portfolio problem on the liability-side of the balance sheet is specified in equations (6) - (10). The theoretical framework for estimating the systematic risk of non-traded assets held by insurance firms was presented by Fairley (1979). Not only did the Fairley results became the foundation for rate application problems at the state level, but the theoretical underpinnings of the model help to formalize the insurance line diversification problem. Hill and Modigliani (1987) presented a detailed case in the state of Massachusetts. More recently, Lee and Cummins (1998) investigated the appropriateness of alternative models for estimating risk of non-traded property and liability insurers. Cummins and Phillips (2003) find that it is important to use firm and line specific risk estimates as the cost of capital varies significantly by line of insurance and varies between large and small insurers. The model presented here is illustrative; hence, a discussion on the estimation methods needed to observe individual parameters is beyond the scope of this paper. Importantly, we note the Cummins and Phillips findings on the potential appropriateness of the Sharpe diagonal model given an adjustment for size and financial distress. We present in equations (6) through (10) the diversifiable components are the *M* underwriting lines where $c_j$ is the fraction issued in underwriting line *j*, and $R_{I_p}$ is the managerially determined desired return from the insurance underwriting portfolio.

$$\sum_{j=1}^{M+1} \sigma_{I_j}^2 c_i - d_6^+ = 0 \tag{6}$$

$$\sum_{j=1}^{M} \beta_{I_j} c_j = \beta_{I_p} \tag{7}$$

$$\sum_{j=1}^{M} c_j = 1 \tag{8}$$

$$\sum_{j=1}^{M} r_{I_j} c_j + d_9^- - d_9^+ = R_{I_p} \tag{9}$$

$$C_{r_I} = \sum_j c_j r_{I_j} \tag{10}$$

### 3.5 ***Premium Flows***

Premium flows for *j-th* underwriting line are captured by $P_j$. Equations (11) through (18) are formulated as managerial goals. Total premiums written are defined by $P_9$, in equation (19).

$$P_1 + d_{11}^- - d_{11}^+ = C_1 P_9 \tag{11}$$

$$P_2 + d_{12}^- - d_{12}^+ = C_2 P_9 \tag{12}$$

$$P_3 + d_{13}^- - d_{13}^+ = C_3 P_9 \tag{13}$$

$$P_4 + d_{14}^- - d_{14}^+ = C_4 P_9 \tag{14}$$

$$P_5 + d_{15}^- - d_{15}^+ = C_5 P_9 \tag{15}$$

$$P_6 + d_{16}^- - d_{16}^+ = C_6 P_9 \tag{16}$$

$$P_7 + d_{17}^- - d_{17}^+ = C_7 P_9 \tag{17}$$

$$P_8 + d_{18}^- - d_{18}^+ = C_8 P_9 \tag{18}$$

$$P_9 = \sum_{j=1}^{M} P_j \tag{19}$$

### 3.6 ***Premium Sub-Totals***

Equations (20) and (21) also capture total premium flows. $P_{10}$ as defined in equation (20) measures total **P**-line premiums and, similarly, equation (21) measures total **non P**-line premium flows. These premium sub-totals are used to facilitate the calculation of two IRIS-defined ratios. The IRIS 'Overall Ratios' dimension causes a need to calculate the net premiums written to surplus ratio as well as the change in writings ratio. It should be noted that within the IRIS dimension, the increase (decrease) in net premiums written, is also a focal point as it is generally expressed as a ratio to net premiums written in the prior year. This latter calculation is easily computed upon solving the NLGP model.

$$P_{10} = P_1 + P_2 + P_3 + P_4 + P_8 \tag{20}$$

$$P_{11} = P_5 + P_6 + P_7 \tag{21}$$

### 3.7 ***The Balance Sheet***

The accounting relationships that define the firm's balance sheet are specified in equations (22) to (37). The ending period account values for the asset side of the balance sheet are specified through equation (26). The ending period values represent the amount of beginning period values that did not mature plus purchases less any sales of the asset. Restrictions on purchases are stated in equations (27) to (31). The model does not permit short sales; the relationship is stated in equations (32) to (36). The accounting identity is stated in equation (37).

$$A_2 = A_2'(1-a) + B_2 - S_2 \tag{22}$$

$$A_3 = A_3'(1-b) + B_3 - S_3 \tag{23}$$

$$A_4 = A_4'(1-c) + B_4 - S_4 \tag{24}$$

$$A_5 = A_5'(1-d) + B_5 - S_5 \tag{25}$$

$$A_6 = A_6'(1-e) + B_6 - S_6 \tag{26}$$

$$B_2 \le f \tag{27}$$

$$B_3 \le g \tag{28}$$

$$B_4 \le h \tag{29}$$

$$B_5 \le k \tag{30}$$

$$B_6 \le l \tag{31}$$

$$S_2 \le A_2'(1-a) \tag{32}$$

$$S_3 \le A_3'(1-b) \tag{33}$$

$$S_4 \le A_4'(1-c) \tag{34}$$

$$S_5 \le A_5'(1-d) \tag{35}$$

$$S_6 \le A_6' \tag{36}$$

$$\sum_i A_i = \sum_j L_j \tag{37}$$

## 3.8 ***Capital / Policyholders' Surplus***

Equation (38) defines the policyholders' surplus account. For the model developed in this research, all investment securities' accounts are treated as perpetuities. Although this

assumption is somewhat unrealistic, it does not interfere with the purpose of the model and its exposition. The managerial goal associated with this account is to minimize under-achieving accumulations to the capital account. In this model policyholders' surplus is defined as a function of the beginning period value plus changes in premiums written. Added to this are the planning period changes in balance sheet values. For example, the equation includes revenue for the fraction of existing government securities that did not mature, $r_2'(1\text{-}a)A_2'$, plus revenue associated with the purchase of new securities, $r_2B_2$, plus (less) capital gains associated with security sales, $[r_2 - (r_2'/r_2) + 1]S_2$, where, again, investment securities are all treated as perpetuities. Except for the common stock account, the revenue structure presented above is similarly applied to all remaining asset side accounts. Income from the common stocks is calculated by adjusting the desired return $R_{S_p}$ by under (over) achievement $-d_4^-$ $(+d_4^+)$ as applied to the end of period dollar commitment to the end of period common stock dollar investment, $A_6$. Finally, the account definition concludes by recording accumulated losses from the fraction of the change in loss-reserves (1-$\alpha_i$) that is charged to the policyholders' surplus account for each of the four loss-reserve accounts. Stated differently, the parameter $\alpha_i$ is the fraction of the change in loss-reserve that is carried forward.

$$L_{10} - d_{37}^- = L_{10}' - E\left(0.62P_{10} + 0.69P_{11}\right) + r_2'(1-a)A_2' + r_2A_2 -$$

$$\left[r_2 - \left(r_2'/r_2\right) + 1\right]S_2 + r_3'\left(1-b\right)A_3' + r_3B_3 -$$

$$\left[r_3 - \left(r_3'/r_3\right) + 1\right]S_3 + r_4'\left(1-c\right)A_4' + r_4B_4 -$$

$$\left[r_4 - \left(r_4'/r_4\right) + 1\right]S_4 + r_5'\left(1-d\right)A_5' + r_5B_5 -$$

$$\left[r_5 - \left(r_5' \middle/ r_5\right) + 1\right] S_5 + \left[\left(R_{S_P} - d_4^- + d_4^+\right)\left(A_6\right)\right] - \left(1-\alpha_1\right)\left(L_5 - L_5'\right) -$$

$$\left(1-\alpha_2\right)\left(L_6 - L_6'\right) - \left(1-\alpha_3\right)\left(L_7 - L_7'\right) - \left(1-\alpha_4\right)\left(L_8 - L_8'\right) \qquad (38)$$

## 3.9 ***Regulatory and Managerial Constraints***

### 3.9.1 *Liquidity*

Equation (39) implements a basic regulatory requirement. This equation requires the dollar investment in federal, state, municipal, and corporate bonds to equal total loss reserve for all insurance lines with active premium flows.

$$A_2 + A_3 + A_4 + d_{39}^- - d_{39}^+ = L_2 + L_3 + L_4 + L_5 \qquad (39)$$

Equations (40) and (41) express the bounds on cash holdings as managerial goals. Taken together, these two equations seek to produce an end-of-period cash account that falls between a high and low dollar range. Both the upper- and lower-bound on cash holding are defined by taking a percent of the total loss reserve. The upper bound is computed against loss reserves by applying the upper bound percent, $\eta_H$. Similarly, the lower bound is computed by applying the lower bound percent $\eta_L$.

$$A_1 + d_{40}^- - d_{40}^+ = \eta_H \left(L_2 + L_3 + L_4 + L_5\right) \qquad (40)$$

$$A_1 + d_{41}^- - d_{41}^+ = \eta_L \left(L_2 + L_3 + L_4 + L_5\right) \qquad (41)$$

3.9.2 *Reserves*

The development of total reserves is accomplished by presenting the loss reserve accounts separately from the unearned premium accounts. Each is detailed below.

3.9.2.1 *Loss Reserve Accounting*

The property-liability company must treat the relationship between current and prior year loss reserves for each individual insurance line against which premiums are written. Because the loss settlement history differs across the *j* lines, the value assigned to the premium roll-off factor, $\delta_j$, is set to reflect this history. A small (large) value is more likely for a long (short) settlement period. Additionally, end-of-period loss reserves also include the portion of premium income earmarked for loss payments throughout the planning period, $\tau_j$. For some lines very little premium income is ever used to pay for losses; hence, only a small fraction of premium income in that line need be set-aside in a reserve. Equations (42) through (45) define this relationship for the loss reserve for underwriting activity in non-P lines, miscellaneous liability, workers compensation liability and automobile liability insurance. We note that the economic relationship presented here is influenced by the one-year reserve development to surplus IRIS ratio.

$$L_2 - d_{42}^{+} = \delta_1 L_2' + \tau_1 P_{10} \tag{42}$$

$$L_3 - d_{43}^{+} = \delta_2 L_3' + \tau_2 P_6 \tag{43}$$

$$L_4 - d_{44}^{+} = \delta_3 L_4' + \tau_3 P_7 \tag{44}$$

$$L_5 - d_{45}^{+} = \delta_4 L_5' + \tau_4 P_5 \tag{45}$$

3.9.2.2 *Unearned Premium Reserve Accounting*

Equations (46) - (49) detail the behavior of unearned premium reserve (UPR). The UPR is the liability item that reflects premium revenues received for contractual coverage not yet fully provided. The ending period UPR account is directly related to a fractional amount of the change in premium flows across two consecutive planning periods. Within the context of a one-period model where premium flows are assumed to accrue evenly throughout the planning period, a predetermined fraction of 0.5 with equal premium flows from one year to the next would result in a constant UPR account as each dollar paid in losses would be replaced by current premium writings.

$$L_6 - d_{46}^{+} = L_6' + 0.5\left(P_{10} - P_{10}'\right) \tag{46}$$

$$L_7 - d_{47}^{+} = L_7' + 0.5\left(P_6 - P_6'\right) \tag{47}$$

$$L_8 - d_{48}^{+} = L_8' + 0.5\left(P_5 - P_5'\right) \tag{48}$$

$$L_9 - d_{49}^{+} = L_9' + 0.5\left(P_5 - P_5'\right) \tag{49}$$

3.9.3 *Kenny Ratio*

The Kenny ratio is implemented with two equations, (50) and (51). Equation (50) is a constraint and equation (51) is a goal. Owing to remarkable profits over the recent decade, the property-casualty industry is known to be overly capitalized. Generally, the Kenny ratio is implemented to scale the premiums to surplus ratio to 2:1. When the ratio exceeds 3:1 the IRIS system is favored for regulatory review. Equation (50) captures the upper bound ($\kappa_U$) on premium writings while equation (51) specifies the lower bound ($\kappa_L$) as a goal which may be either under- or over-achieved.

$$P_9 + d_{50}^- = \kappa_U L_{10} \tag{50}$$

$$P_9 + d_{51}^- - d_{51}^+ = \kappa_L L_{10} \tag{51}$$

3.9.4 *Profitability*

3.9.4.1 *Maximization of Firm Value*

Equation (52) reflects the maximization of firm value goal. The model states this objective by seeking to maximize the value of the policyholders' surplus account. By setting the right-hand-side of this goal equation to an arbitrarily high value, the NLGP model will produce a solution that reflects the most efficient allocation of firm resources within the context of the defined financial relationships.

$$L_{10} + d_{52}^- = 10^{37} \tag{52}$$

3.9.4.2 *Contemporary Objective: Governance and Agency Costs*

Recent research suggests that managers may act in their own best interests when un-monitored (for a more formal discussion of agency theory see Jensen and Meckling (1976)). The discipline imposed by financial markets often operates too slow to what could be excessive deterioration in the benefits that accrue to the owners. Market discipline alone may not be sufficient to assure an efficient utilization of firm resources. For example, property-liability insurers routinely face the question of whether to rely upon the independent agency system or exclusive dealing insurers (tied agent). Because independent agents own the rights to their customer list, the agent has the unrestricted right to place business with any number of competing insurers. Regan (1999) provides a comprehensive retrospective on the hypothesis that the threat to move customers allows the agent to command higher compensation, thereby producing higher expense ratios for independent agency insurers. Conventional wisdom would suggest, as observed for personal line insurers, that market share would decline for the

independent agency system. Instead, recent research has shown that market share has remained relatively stable despite these higher operating costs.

Under the traditional application of firm agency theory it is the responsibility of managers to set firm ALM objectives while it is the responsibility of owners to set specific constraints that will protect their own interests. Since the tied agent's revenue is produced by a single insurer, in the case of a significant adverse experience with a particular underwriting line the tied agent would desire to demand a relatively higher payment for bearing the risk. Since this course of action may not be available to the agent, an alternative would be to focus on less complex underwriting lines (see Regan (1997) for additional discussion). However, this latter choice may be in conflict with the goals of owners who seek an efficient portfolio of underwriting business.

Equation (53) provides a definition of the agency conflict created by the desire of firm managers to increase income through aggressive asset-side management while limiting exposure to high yielding but high risk underwriting lines. The model defines an overall return that is a function of the return to both the common stock and underwriting portfolios. Specifically, utilize the two efficient portfolio sub-decisions captures a firm-profitability objective for firm financial managers. This equation provides a mechanism to unite the goals of managers and owners.

$$C_{r_I} + C_{r_s} + d_{53}^{-} - d_{53}^{+} = R_A \qquad (53)$$

### 3.10 ***The Hierarchical Objective Function***

Three hierarchical priorities define the objective function of the property-liability NLGP model. The flexibility of the NLGP model is characterized by the specification of the objective function. This specification will vary at the firm level. The statement provided here is but one of many different characterizations.

The first priority ($Z_1$), which must be achieved before consideration of any lower level priorities, minimizes under-achievement of two sub-objectives. Both sub-objectives have equal weight within this priority level. Specifically, rate of return objectives for the security portfolio and the insurance lines portfolio are specified, respectively. The second priority ($Z_2$) seeks to minimize over-achievement in the portfolio risk dimension. When taken together, these two objectives operate to fix the specified rate of return and then find the optimal allocation of investment-assets and operating lines at the respective associated rate of return. The third objective ($Z_3$) represents the traditional goal of shareholder maximization.

$$Min: Z_1\left[d_4^- + d_9^-\right] + Z_2\left[d_1^+ + d_6^+\right] + Z_3\left[d_{52}^-\right] \qquad (0)$$

## 4. Data

Booth and Besseler (1989) as well as Dash and Kajiji (2002) have effectively modeled stochastic bank ALM models by the use of a hypothetical economic scenario. To demonstrate the contribution of the property-liability NLGP model developed above, we follow earlier efforts and introduce an appropriate hypothetical economic scenario. To bring a degree of realism to the scenario we produce a scaled hypothetical financial statement with a $10,000 base by

averaging reported values across U.S. based property-liability insurers. Individual account balances are created by use of the historical percentages as observed in the averaged statement. Within this context, beginning period values are presented in table 4.

<<< Insert Table 4>>>

The NLGP model specification is deterministic. To solve the model the decision-maker must provide forecast estimates on several key variables. Appropriate methodologies for bank ALM forecasting is developed elsewhere (for examples, see Dash and Kajiji (2002), Kosmidou and Zopounidis (2002), and Booth and Bessler (1989)). Relevant methods for insurance ALM models are discussed by Yu, et. al. (2003), Consiglio, et. al. (2002), Wang and Yang (2002), and Cummins and Nye (1981). Further, to examine the robustness of model, we focus on solutions over a range of alternative forecast estimates made for the growth in premium volume. Premium volume growth from the beginning period value is allowed to vary from a decline of 98% to an expansion of 100%. The individual forecast values are presented in table 5.

<<< Insert Table 5>>>

The model requires bank management to provide a forecast value for security purchases over the planning period. That is, the model will attempt to buy securities as long as securities are more profitable than the cost associated with the underwriting function. Restricting accumulations to no more than the forecasted value controls accumulations to various accounts. Additionally, the model does not have a specification that takes into account investment grade rankings. To capture the effect of investment grade ranking an upper bound is specified for the purchase of new securities (indicated by the maximum percent increase). Finally, where

appropriate, the maturation parameter is stated for each account category. This parameter captures the percentage of the asset that matures, or converts to cash, over the planning horizon.

<<< Insert Table 6>>>

## 4.1 ***Interest Rate Parameters***

Interest rate parameters needed to specify revenue flows are presented in table 7. Variables with primes represent beginning period values. Variables without primes represent the expected rates over the course of the planning horizon.

<<< Insert Table 7>>>

## 4.2 ***Policy Parameters***

Model data for policy parameters are presented in table 8. These values are observed directly from regulatory statues or are determined explicitly by firm management history. All notations are as previously defined in section 3.

<<< Insert Table 8>>>

## 4.3 ***Security and Insurance Line Portfolio Parameters***

The data presented in table 9 is required for application of the Sharpe diagonal portfolio optimization problem. The table is divided into two sections. Data that specifies the characteristics of each investment security may be found in the left-most section of the table. This data includes each securities expected rate of return, its systematic risk coefficient (beta),

and a measure of unsystematic risk ($\sigma$). The same data is presented for each of the modeled insurance lines. In the case of individual securities, it is a trivial matter to extend the specification to include a larger and more diverse group of securities. However, the data for non-traded assets will require more attention to the historical performance of these assets as they relate to the risk level of the firm. A detailed state-specific case example and accompanying theoretical discussion may be found in Hill et. al. (1987). A more comprehensive review of the pricing issue is provided by Ericssony and Reneby (2002).

<<< Insert Table 9>>>

## 5. The Model Solution

The results of solving the model using the preceding data with a two-grid point approximation of all functions are presented in the tables 10 through 13. A comparison of the ending period and beginning period balance sheets are generated by varying two policy dimensions – one on each side of the balance sheet. We examine a range of alternative required returns for the efficient security portfolio on the asset side and alternative growth rate scenarios for insurance underwriting premiums on the liability side. These results are presented in table 10 and table 11, respectively. The optimal solution to the two portfolio problems is presented in table 12 and figure 1. It should be noted that for solution purposes an additional set of constraints was added to the optimization problem for securities (not shown above). These constraints required at least a two percent investment in each security. These lower-bound constraints were needed to accommodate the coarseness of the small separable programming grid point approximation for solution variables with decimal point answers between zero and

one. The results section concludes with a presentation and analysis of the premium flows produced by solving the NLGP model.

<<< Insert Table 10, 11>>>

## 5.1 ***Agency Theoretic Considerations***

In addition to solving the model for each of the different premium growth rates, the agency rate of return (equation 53) was also varied as a way of examining the effect of this dimension on the optimal solution values reported for the two embedded portfolio problems. More emphasis was devoted to solutions characterized by an eight percent growth rate in premiums. At this growth level for premiums, the total portfolio (agency) rate of return objective was set to the following values: 8, 12, 15, 18, and 50 percent, respectively.

## *5.2* ***Optimal Diversification***

### 5.2.1 *Common Stock Diversification*

Given premium growth of 8% and a total portfolio rate of return of 8%, the ending period value of the common stock account is $602.80 (see table 10). The equity diversification strategy within the common stock account is presented in table 12. The diversification strategy emphasizes two securities. These are security 2 at 13.47% and security 7 at 72.53%. All other securities enter at the managerially constrained 2% level. The return generated by the equity investments with this pattern of diversification is 6.59%.

<<< Insert Table 12 >>>

### 5.2.2 *Operating Line Diversification*

The rate of return from the insurance underwriting function is 1.3%. The normative results report a return of 6.59% and 1.30% for the securities account and the insurance portfolio, respectively. The total of 7.9% differs from the target of 8% solely due to rounding error given a two-grid point approximation. Interestingly, given the first level priority to minimize under-achievement of the expected return goal for both the securities account and the operating lines portfolio, the optimal solution for the operating lines generated a positive rate of return of 1.30%. This return level was generated while meeting all other constraints and goals as specified for the underwriting function. Figure 1 presents two efficient frontiers. The top-most efficient frontier represents the risk-return combinations available with an 8% growth in premiums given optimal investments in both investment equities and the underwriting lines. The second efficient frontier shows the equity risk-return combinations for comparison. Figure 1 also shows three points labeled B, K, and M. These points represent the risk-return relationships of the insurance portfolios alone. Stated differently, without the ability to invest in asset-side equity investments, property-liability firms would generate risk-return profiles at these three interior points.

<<< Insert Figure 1 >>>

## 5.3 ***Premium Volume***

At the 8% growth level, the model does not take all of the premiums that were forecasted. This is directly related to the linkage of premium income to optimal insurance line investment. The under (over)-achievement of each account is reported in table 13a and table 13b. Negative values suggest a failure to take premiums in an amount equal to the optimal contribution.

Positive values, by contrast, suggest over-achievement of the optimal flow goal; that is, more premiums were written than the optimal allocation recommended.

<<< Insert Tables 13a and 13b >>>

## 5.4 ***Regulatory Objectives***

Cash holdings also meet the bounds established in equations (39) through (41). All loss-reserve and unearned premium reserve accounting are also met. Importantly, the NLGP specification lends itself to important ratio calculations. Presented in table 14 are commonly computed solvency ratios. The values reported by these ratios reflect the artificial nature of the generalized balance sheet. Nevertheless, the impact of the optimizing process is evident. For example, at the 8% growth rate in premiums, the Kenny ratio is quite conservative. The ratio increased slightly when compared to the beginning period economic characterization; however, the increase is a small 0.04 points. This value, of course, is highly dependent on the beginning period value of policyholders' surplus.

Other important trends are also visible. The 1-year change in premium writings as a ratio to policyholders' surplus shows a models decline. These trends, as well as others reflected in the ratio analysis, show that the model was able to produce desired profitability while simultaneously increasing the efficiency of allocated resources.

<<< Insert Table 14 >>>

### 5.5 ***Other Considerations***

The model suggests that the property-liability company should avoid any sale of assets to raise liquidity. Instead, management should focus on generating cash flow through premium-based activity. When it is no longer possible to invest these funds in traditional investment categories, then management should search out "other assets" in which to place funds generated through the issuance of insurance claims.

Although the topic was not explored fully in this paper, the NLGP model developed above could logically be extended to generate an efficient frontier of alternative balance sheet opportunities. Such an extension would add another useful dimension to the overall planning process. For example, with such an extension management would be able to explore the *price of risk* associated with the trade-off between investment and underwriting opportunities. In addition to providing a balance-sheet plan, management would gain an increased understanding of how subjectively determined ordinal rankings of firm priorities are likely to affect the risk-adjusted return available to shareholders. To regulators, such an exercise would provide useful information about the depth of management control over total risk. This information would assist regulators in determining the range of feasible alternative balance sheet combinations that are actually available to management.

## 6. Summary and Conclusion

The joint purpose of this paper is to present a NLGP model based upon the separable programming method that is suitable for solving multiple mean-variance efficiency frontiers in hierarchical balance sheet optimization problem faced by post FIMA property and liability

insurers. The NLGP model developed in this paper extends previous model-building efforts by permitting the specification of multiple hierarchical goals and the simultaneous treatment of optimality in both the investment and underwriting functions. The results produced for the generalized property and liability firm and its hypothetical financial statement are consistent with prior findings and clearly demonstrated the usefulness of the direct and explicit treatment of the dual portfolio problem inherent in property-liability ALM planning.

The NLGP method requires a linearization of the nonlinear problem. The number of grid points used in the initial transformation determines the refinement of the linearization. As the number of grid points increases, the size of the problem also increases. In this paper the minimum of two grid points was invoked in the linearization scheme. Except for the percentage investment in securities, the accuracy of the solution appears adequate for the model specification. It is possible that increasing the separable programming grid points may generate a slightly more robust solution; however, it will come with the penalty of increased solution time. On average, the models reported in this paper required 2:39 minutes on an IBM PC with a 1Ghz AMD Athlon class processor.

Future extensions are needed in at least two areas. First, a useful model extension would link the acquisition of premium flows to the optimal allocation derived in the insurance-line portfolio problem. The implementation of this model extension would require additional model decomposition as it invokes a cross product specification (optimal percent times total premiums). To invoke the NLGP algorithm, this term must be 'separated.' The separation of these terms will add new variables and new constraints for each cross-product term. The impact is two-fold.

A more detailed model specification is achieved, but it will come at the cost of increased solution time.

Second, the robustness of the current solution suggests a need to enhance the economic specification of the current model. For example, the model ignores tax effects, derivative instruments, timing decisions, and a more detailed profit-loss functional definition. The latter point is of keen interest as a detailed profit-loss statement would permit a greater focus on important ratios that are not fully enumerated in the current model (e.g., the combined ratio). Each of these factors has been considered elsewhere in the literature. By incorporating these two extensions, the NLGP approach to building ALM models in the property-liability industry appears to be sound and fruitful.

**Table 1. Generalized Property-Liability Firm Financial Statement**

| *Assets* | *Beginning Period Values* | *Purchase Variable* | *Sale Variable* | *Rate Parameter* | *Liabilities* | *Beginning Period Values* |
|---|---|---|---|---|---|---|
| Cash | $A_1'$ | | | | Liabilities & Other | $L_1'$ |
| Federal Govt. Sec. | $A_2'$ | $B_2$ | $S_2$ | $r_2$ | Loss Reserves: **P** Lines | $L_2'$ |
| State & Local Sec. | $A_3'$ | $B_3$ | $S_3$ | $r_3$ | Loss Reserves: Misc. | $L_3'$ |
| Corp. Bonds | $A_4'$ | $B_4$ | $S_4$ | $r_4$ | Loss Reserves: Workers' Comp. | $L_4'$ |
| Preferred Stock | $A_5'$ | $B_5$ | $S_5$ | $r_5$ | Loss Reserves: Auto-Bodily Injury | $L_5'$ |
| Common Stock | $A_6'$ | $B_6$ | $S_6$ | $r_6$ | Unfunded Premiums: **P** Lines | $L_6'$ |
| Other Assets | $A_7'$ | $B_7$ | $S_7$ | $r_7$ | Unfunded Premiums: Misc. | $L_7'$ |
| | | | | | Unfunded Prem. Res: Workers' Comp. | $L_8'$ |
| | | | | | Unfunded Prem. Res: Auto-Bodily Injury | $L_9'$ |
| | | | | | Policyholders' Surplus | $L_{10}'$ |

**Table 2. Insurance Lines and their Premium**

| ***Insurance Line*** | ***Property Premium*** | ***Non-Property Premium*** |
|---|---|---|
| Fire | $P_1$ | |
| Allied Fire Liability | $P_2$ | |
| Homeowners' Liability | $P_3$ | |
| Commercial-Multi Peril | $P_4$ | |
| Automobile Liability | | $P_5$ |
| Miscellaneous Liability | | $P_6$ |
| Workers' Compensation | | $P_7$ |
| Automobile Physical Damage | $P_8$ | |

**Table 3. Modeling Parameters and Variables**

| ***Parameter*** | ***Variable*** | ***Description*** |
|---|---|---|
| N | | Number of investment securities |
| M | | Number of insurance lines |
| $\sigma_{S_i}$ | | Unsystematic risk for the *i-th* investment security (S) |
| $\sigma_{I_j}$ | | Unsystematic risk for the *j-th* insurance line (I) |
| $r_{S_i}$ | | Rate of return on the *i-th* investment security |
| $r_{I_j}$ | | Rate of return on the *j-th* insurance line |
| $\beta_{S_i}$ | | Beta for the *I-th* investment security |
| $\beta_{I_j}$ | | Beta for the *j-th* insurance line |
| | $x_i$ | Fraction invested in the *i-th* security |
| | $c_j$ | Fraction invested in the *j-th* insurance line |
| a | | Maturation fraction of federal government securities |
| b | | Maturation fraction of state and local obligations |
| c | | Maturation fraction of corporate bonds |
| d | | Maturation fraction of preferred stock |
| e | | Maturation fraction of common stock |
| f | | Purchase forecast of federal government securities |
| g | | Purchase forecast of state and local obligations |
| h | | Purchase forecast of corporate bonds |
| k | | Purchase forecast of preferred stock |
| l | | Purchase forecast of common stock |
| $\alpha_i$ | | Change in policyholders' surplus |
| $\delta_j$ | | The fraction of loss reserve of the *j-th* insurance line carried over to the next year |
| $\tau_j$ | | The fraction of premiums kept as reserve for the *j-th* insurance line |
| $\kappa$ | | Kenny ratio parameter (see definition in sec 3.9.3) |
| $\eta$ | | Bounded Liquidity parameter (see definition in sec 3.9.1) |
| p | | Subscript referring to the portfolio |

**Table 4. Beginning Period Values**

| *Assets* | *Value* | *Liabilities* | *Value* |
|---|---|---|---|
| Cash | 947 | Liabilities & Other | 958 |
| Federal Govt. Sec. | 1,043 | Loss Reserves: P Lines | 2,299 |
| State & Local Sec. | 4,187 | Loss Reserves: Misc. | 584 |
| Corp. Bonds | 1,261 | Loss Reserves: Workers' Comp. | 638 |
| Preferred Stock | 378 | Loss Reserves: Auto-Bodily Injury | 1,132 |
| Common Stock | 1,507 | Unfunded Premiums: P Lines | 1,015 |
| Other Assets | 677 | Unfunded Premiums: Misc. | 169 |
| | | Unfunded Prem. Res: Workers' Comp. | 285 |
| | | Unfunded Prem. Res: Auto-Bodily Injury | 479 |
| | | Policyholders' Surplus | 2,441 |
| **TOTAL** | $10,000 | **TOTAL** | $10,000 |

**Table 5. Premium Dollar Values: Beginning Period and Various Forecast Levels**

| *Insurance Lines* | *Beginning Period* | *-25%* | *25%* | *30%* | *50%* | *100%* |
|---|---|---|---|---|---|---|
| Fire | 461.30 | 345.98 | 576.63 | 599.69 | 691.95 | 922.60 |
| Allied Fire Liability | 117.50 | 88.13 | 146.88 | 152.75 | 176.25 | 235.00 |
| Homeowners' Liability | 277.70 | 208.27 | 347.12 | 361.01 | 416.55 | 555.40 |
| Commercial-Multi Peril | 243.00 | 182.25 | 303.75 | 315.90 | 364.50 | 486.00 |
| Automobile Liability | 433.20 | 324.90 | 541.50 | 563.16 | 649.80 | 866.40 |
| Miscellaneous Liability | 37.90 | 28.43 | 47.38 | 49.27 | 56.85 | 75.80 |
| Workers' Compensation | 41.40 | 31.05 | 51.75 | 53.82 | 62.10 | 82.80 |
| Auto. Physical Damage | 77.00 | 57.75 | 96.25 | 100.10 | 115.50 | 154.00 |

**Table 6. Asset / Liability Forecast with Corresponding Maturation**

| *Asset Category* | *Upper Bound of Percent Increase* | *Forecast Parameter* | *Maturation Parameter* |
|---|---|---|---|
| Federal Govt. Sec. | 30% | f = 1,355.90 | a = 0.80 |
| State & Local Sec. | 15% | g = 4,815.05 | b = 0.05 |
| Corp. Bonds | 50% | h = 630.50 | c = 0.00 |
| Preferred Stock | 30% | k = 113.40 | d = 0.00 |
| Common Stock | 40% | l = 602.80 | |

**Table 7. Interest Rate Parameters**

| **Beginning Period** | **Rate** | **Planning Period** | **Rate** |
|---|---|---|---|
| $\mathbf{r}_2'$ | 0.060 | $\mathbf{r}_2$ | 0.070 |
| $\mathbf{r}_3'$ | 0.050 | $\mathbf{r}_3$ | 0.054 |
| $\mathbf{r}_4'$ | 0.070 | $\mathbf{r}_4$ | 0.073 |
| $\mathbf{r}_5'$ | 0.065 | $\mathbf{r}_5$ | 0.069 |

**Table 8. Policy Parameters**

| **Parameter** | **Value** |
|---|---|
| $\delta_i$ | 1.47 |
| $\tau_1$ | 0.15 |
| $\tau_2$ | 1.54 |
| $\tau_3$ | 1.54 |
| $\tau_4$ | 1.47 |
| $\alpha_i$ | 0.20 |
| $\kappa_H$ | 4.00 |
| $\kappa_L$ | 2.00 |
| $\eta_H$ | 0.15 |
| $\eta_L$ | 0.10 |

**Table 9. Portfolio Model Data**

| *Security* | *E(r)* | *Beta* | $\sigma^2$ | *Insurance Line* | *E(r)* | *Beta* | $\sigma^2$ |
|---|---|---|---|---|---|---|---|
| Sec_001 | 0.064 | 0.896 | 0.0266 | Fire | 0.010 | 0.850 | 0.0013 |
| Sec_002 | 0.062 | 0.468 | 0.0072 | Allied Fire Liability | 0.050 | 1.895 | 0.0380 |
| Sec_003 | 0.146 | 1.110 | 0.0441 | Homeowners' Liability | -0.001 | 0.350 | 0.0270 |
| Sec_004 | 0.173 | 1.361 | 0.0292 | Commercial-Multi Peril | 0.068 | 1.361 | 0.0901 |
| Sec_005 | 0.198 | 1.273 | 0.0756 | Automobile Liability | -0.010 | 0.857 | 0.0035 |
| Sec_006 | 0.056 | 0.590 | 0.0310 | Miscellaneous Liability | -0.014 | 1.313 | 0.1500 |
| Sec_007 | 0.128 | 0.527 | 0.0207 | Workers' Compensation | 0.028 | 0.981 | 0.0091 |
| Sec_008 | 0.190 | 1.733 | 0.0384 | Auto. Physical Damage | 0.004 | 0.568 | 0.0280 |
| Sec_009 | 0.116 | 1.041 | 0.0437 | | | | |
| Market Index | | | 0.0397 | Insurance Industry Index | | | 0.0800 |

**Table 10. Projected Growth Rates and Optimal Asset Allocation Strategies**

| *Projected Premium Growth Rates* | *E(r) Total Portfolio* | *Cash* | *Federal Govt. Securities* | *State & Local Securities* | *Corporate Bonds* | *Preferred Stock* | *Common Stock* | *Other Assets* | *Total Assets* |
|---|---|---|---|---|---|---|---|---|---|
| -98.00% | 1.00% | 944.20 | 1355.90 | 4875.18 | 1891.50 | 113.40 | 602.80 | 708.15 | 10491.14 |
| -25.00% | 1.00% | 1052.74 | 1355.90 | 6521.71 | 1261.00 | 113.40 | 602.80 | 789.55 | 11697.10 |
| 0.05% | 18.00% | 1313.62 | 1355.90 | 7955.30 | 1891.50 | 491.40 | 602.80 | 985.21 | 14595.73 |
| 8.00% | 8.00% | 1093.06 | 1355.90 | 6268.70 | 1891.50 | 113.40 | 602.80 | 819.80 | 12145.17 |
| 8.00% | 15.00% | 1156.66 | 0.00 | 7955.30 | 1891.50 | 378.00 | 602.80 | 867.49 | 12851.75 |
| 4.00% | 18.00% | 1496.88 | 1564.50 | 7955.30 | 1891.50 | 491.40 | 3109.80 | 1122.66 | 16632.05 |
| 8.00% | 18.00% | 1156.66 | 0.00 | 7955.30 | 1891.50 | 378.00 | 602.80 | 867.49 | 12851.49 |
| 15.00% | 18.00% | 1140.83 | 208.60 | 7955.30 | 630.50 | 378.00 | 1507.00 | 855.62 | 12675.62 |
| 25.00% | 18.00% | 1496.88 | 1564.50 | 7955.30 | 1891.50 | 491.40 | 2109.80 | 1122.66 | 16632.05 |
| 30.00% | 18.00% | 1117.11 | 1355.90 | 6115.78 | 1891.50 | 491.40 | 602.80 | 837.83 | 12412.32 |
| 50.00% | 18.00% | 1156.96 | 1355.90 | 6866.78 | 1891.50 | 113.40 | 602.80 | 867.72 | 12855.06 |
| 100.0% | 18.00% | 1228.83 | 1355.90 | 7539.62 | 1891.50 | 113.40 | 602.80 | 921.62 | 13653.68 |
| 8.00% | 12.00% | 1156.66 | 0.00 | 7955.30 | 1891.50 | 378.00 | 602.80 | 867.49 | 12851.75 |
| 8.00% | 50.00% | 1496.88 | 1564.50 | 7955.30 | 1891.50 | 491.40 | 2109.80 | 1122.66 | 16632.05 |

**Table 11. Projected Growth Rates and Optimal Liability Allocation Strategies**

| *Projected Premium Growth Rates* | *E(r) Total Portfolio* | *Liabilities and Other* | *Loss Reserve P Lines* | *Loss Reserve Misc.* | *Loss Reserve Workers Comp* | *Loss Reserve Auto B.I.* | *Un-earned Premium P Lines* | *Un-earned Premium Misc.* | *Un-earned Premium Workers Comp* | *Un-earned Premium Auto Bodily Injury* | *Policy-holders Surplus* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -98.00% | 1.00% | 996.66 | 2302.53 | 585.17 | 639.28 | 1134.26 | 1615.02 | 188.33 | 306.11 | 521.83 | 2201.95 |
| -25.00% | 1.00% | 1111.22 | 2431.36 | 627.77 | 685.82 | 1216.89 | 2044.44 | 202.16 | 321.23 | 679.95 | 2376.26 |
| 0.05% | 18.00% | 1386.59 | 2475.52 | 642.40 | 701.79 | 1245.25 | 2191.65 | 206.91 | 326.41 | 734.21 | 4685.01 |
| 8.00% | 8.00% | 1153.79 | 2482.53 | 647.04 | 706.86 | 1254.25 | 2215.03 | 208.42 | 328.06 | 751.43 | 2397.77 |
| 8.00% | 15.00% | 1730.16 | 2482.53 | 647.04 | 706.86 | 1254.25 | 2215.03 | 208.42 | 328.06 | 751.43 | 2527.99 |
| 4.00% | 18.00% | 1580.04 | 2479.00 | 644.70 | 704.31 | 1249.72 | 2203.27 | 207.66 | 327.23 | 742.76 | 6493.36 |
| 8.00% | 18.00% | 1730.16 | 2482.53 | 647.04 | 706.86 | 1254.25 | 2215.03 | 208.42 | 328.06 | 751.43 | 2527.99 |
| 15.00% | 18.00% | 1327.57 | 2501.95 | 651.12 | 711.32 | 1262.17 | 2279.74 | 209.74 | 329.51 | 766.59 | 2636.15 |
| 25.00% | 18.00% | 1580.04 | 2506.54 | 656.96 | 717.69 | 1273.49 | 2295.04 | 211.64 | 331.58 | 788.25 | 6270.82 |
| 30.00% | 18.00% | 1179.17 | 2514.84 | 659.88 | 720.88 | 1279.15 | 2322.71 | 212.59 | 332.61 | 760.64 | 2429.86 |
| 50.00% | 18.00% | 1221.23 | 2563.71 | 671.55 | 733.63 | 1301.79 | 2485.63 | 216.37 | 336.75 | 842.40 | 2481.99 |
| 100.0% | 18.00% | 1297.10 | 2651.95 | 700.73 | 765.51 | 1358.38 | 2779.75 | 225.85 | 347.10 | 950.70 | 2576.60 |
| 8.00% | 12.00% | 1730.16 | 2482.53 | 647.04 | 706.86 | 1254.25 | 2215.03 | 208.42 | 328.06 | 751.43 | 2527.99 |
| 8.00% | 50.00% | 1580.04 | 2482.53 | 647.04 | 706.86 | 1254.25 | 2215.03 | 208.42 | 328.06 | 741.18 | 6468.65 |

**Table 12. Solution: Optimal Portfolio Allocations**

| *Security* | $x_i$ | *Insurance Line* | $C_j$ |
|---|---|---|---|
| Sec_001 | 2.00% | Fire | 23.19% |
| Sec_002 | 13.47% | Allied Fire Liability | 6.96% |
| Sec_003 | 2.00% | Homeowners' Liability | 16.44% |
| Sec_004 | 2.00% | Commercial-Multi Peril | 14.39% |
| Sec_005 | 2.00% | Automobile Liability | 29.77% |
| Sec_006 | 2.00% | Miscellaneous Liability | 2.24% |
| Sec_007 | 72.53% | Workers' Compensation | 2.45% |
| Sec_008 | 2.00% | Auto. Physical Damage | 4.56% |
| Sec_009 | 2.00% | | |
| $R_p$ | 6.59% | $R_p$ | 1.30% |
| $\sigma_p$ | 4.19% | $\sigma_p$ | 9.18% |

**Figure 1: Efficient Frontiers**

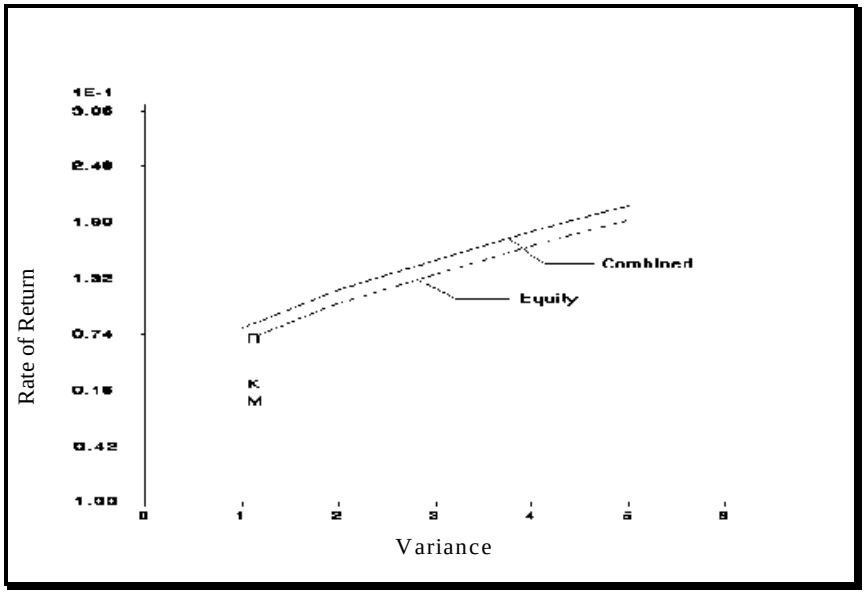


**Table 13a. Difference: Forecast and Optimal Premium Volume**

| ***Growth*** | ***-98%*** | ***-25%*** | ***0.05%*** | ***4%*** | ***8%*** |
|---|---|---|---|---|---|
| Fire | 3.67 | 137.70 | 183.99 | 167.41 | 151.23 |
| Allied Fire | -2.51 | 24.79 | -125.56 | -130.52 | -135.54 |
| Homeowners | -3.11 | -116.62 | -155.87 | -161.72 | -167.94 |
| Coml MP | 4.10 | 153.83 | 205.20 | 213.30 | 221.51 |
| Auto Liab | 7.84 | 293.85 | 392.00 | 407.47 | 423.14 |
| Misc. Liab | -0.78 | -29.32 | -39.12 | -40.66 | -42.23 |
| Worker Comp | 0.67 | 30.90 | 41.32 | 42.95 | 44.61 |
| Auto PD | 1.54 | 57.75 | 77.04 | 80.08 | 83.16 |
| Forecast | 33.78 | 1266.75 | 33.78 | 1756.56 | 1824.12 |
| Actual | 33.78 | 1266.75 | 33.78 | 1733.03 | 1777.06 |

**Table 13b. Difference: Forecast and Optimal Premium Volume (cont.)**

| ***Growth*** | ***15%*** | ***25%*** | ***30%*** | ***50%*** | ***100%*** | ***8%*** |
|---|---|---|---|---|---|---|
| Fire | 333.30 | 142.46 | 148.16 | 275.40 | 367.20 | 151.23 |
| Allied Fire | -144.32 | -156.87 | -163.15 | -188.25 | -251.00 | -135.54 |
| Homeowners | -178.82 | -194.37 | -125.27 | -68.26 | -311.00 | -147.44 |
| Coml MP | 235.87 | 256.38 | 266.63 | 307.65 | 410.20 | 221.51 |
| Auto Liab | 450.57 | 489.75 | 432.46 | 587.70 | 783.60 | 402.64 |
| Misc. Liab | -44.96 | -48.87 | -50.83 | -58.65 | -78.20 | -42.23 |
| Worker Comp | 47.51 | 51.64 | 53.71 | 61.99 | 82.70 | 44.61 |
| Auto PD | 88.55 | 96.25 | 100.10 | 115.50 | 154.00 | 83.16 |
| ***Forecast*** | ***1942.35*** | ***2111.25*** | ***2195.70*** | ***2533.50*** | ***3378.00*** | ***1824.12*** |
| ***Actual*** | ***1942.35*** | ***2024.21*** | ***2028.30*** | ***2533.50*** | ***3378.00*** | ***1756.56*** |

**Table 14. Financial Ratios**

| ***Ratio*** | ***Ending Period*** | ***Beginning Period*** |
|---|---|---|
| Total Prem. to Surplus | 0.74 | 0.69 |
| Liabilities to Liquid Assets | 0.47 | 0.48 |
| Loss Reserves to Surplus | 1.60 | 1.44 |
| Allied Fire to Fire Lines | 0.28 | 0.25 |
| 1 Year Change in Writings to Policyholders' Surplus | 0.05 | 0.08 |
| Kenny Ratio | 1.35 | 1.31 |

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