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Efficient multiple objective neural network mapping of state-wide high school achievement

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Abstract. During the last decade, public school researchers in the U.S. have devoted substantial effort to indentify the factors that most effectively impact the two targeted high school achievement indexes of mathematics (MAI) and English arts (ELA). This paper examines the use of an augmented radial basis function neural network (RANN) to estimate efficient production-theoretic scale estimates for several factors. The research examines all Rhode Island high schools and is based on the application of a truncated second-order translog production function. For comparative scale metrics, alternative results are derived by solving double-log (DL) OLS model estimation. As expected, differing estimates of scale economies were observed across alternative models specifications. For example, the results generated by both models corroborated the conventional wisdom that high school mathematics achievement suffers some percentage decline for any percentage increase in the non-white school population. However, the nonlinear mapping of the RANN amplifies this finding by providing evidence that it is actually the percentage increase in the interaction between non-white student population and students who are eligible for a free or reduced lunch that is most responsible for the observed reduction in MAI scoring performance. Additional new insights are produced across other traditional production factors.

Keywords: neural networks; quantitative policy modeling; analysis of education

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Introduction

The motivation for expenditures on education lies in the belief that the education of children is fundamental to the future economic growth and a lasting democracy. In turn, these outcomes are thought to be the cornerstones upon which a democracy and political stability are built. Contemporary research offers many examples of including education factors in growth models in order to better understand the implications of education policy on macroeconomic performance as well as local and regional socio-economic economics. For example, Barro and Sala-i-Martin (1995) have found that years of secondary and higher education exposure contribute positively toward economic growth. The benefits that are measured as economic growth are also manifested as positive attitudes and discipline as it relates to the workforce and the ability of the workforce to incorporate new technologies (Lucas, (1988)). Rebelo (1991) extended this line of reasoning by introducing physical capital as an additional input. The model

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of Romer (1990), which assumes that the creation of new ideas is a direct function of investment into human capital, clearly ties together a body of thought that argues for the efficient production of education. A number of studies involving small growing economies have examined the relationship between the production embedded in human capital and sources of productivity (Haouas and Yagoubi (2005)). In a slightly different approach, Park (2006) empirically investigated the growth implications of a dispersed population in terms of educational attainment levels. The commonality among these studies and others is an agreement that there is a statistically significant link between human productivity and economic growth.

This paper extends prior model-building efforts directed at uncovering the relative importance of the predictive factors that explain variability in educational achievement at the high school level. Because achievement models are challenging to estimate and tend to lack temporal stability this paper introduces a multiple-objective nonparametric mapping method, the K4 radial basis function (RBF) artificial neural network (ANN), as an alternative approach by which to extract factor elasticity metrics. The paper proceeds as follows. Section 2 presents the economic reasoning supporting the use of a RANN to estimate a double-log production function. Section 3 presents the data and identifies data reduction and transformation related issues. Econometric modeling results are presented in section 4. Section 5 provides a summary and conclusion.

Production economics

Double-log production functions

The double log functional form of the production function is well known for its representation of the functional relationship of an output to factor inputs. A popular form of the double-log model is the well known Cobb-Douglas (1928) function. For our purposes, the generalized double-log model for p inputs is expressed as:

$$f(x) = A\prod_{i=1}^{p} x_i^{\beta_i}, \beta_i > 0, i = 1, 2, \dots, p \tag{1}$$

This functional form has the following properties: a) strict monotonic – if $\forall\, x, x' \in \mathcal{R}_+^n\ x \geq x'$ then $f(x) \geq f(x')$; b) quasi-concavity – $V(y) = [x : f(x) \geq y]$ is a convex set; c) strict essentiality – $f(x_1, x_2, \dots, 0, \dots, x_n) = 0$ for all $x_i > 0$; d) the set $V(y)$ is finite, nonnegative, real valued and single valued for all nonnegative and finite x and thereby continuous and everywhere twice-continuously differentiable; and, e) $f(x)$ is homogenous of degree l = sum of β_i.

In a manner that is consistent with the properties stated above, all inputs can be interchanged without affecting output and each input must be used in strictly positive amounts to obtain a positive output. When applied to many micro level production estimation problems, it is not unusual for the researcher to encounter some observations with zero values. That is, some observations may have a positive level of output but zero use in one or more of the inputs. This would suggest that at some output levels one or more of the zero-valued inputs are non-essential for production. In order to employ a double-log specification (or, the more general translog specification) it becomes necessary to make some transformation to the zero-value arguments.

$$\ln(y) = \ln A + \sum_{i=1}^{p} \beta_i \ln(x_i),\ \beta_i > 0 \tag{2}$$

The transformation of a modified zero-element argument is generally accomplished in one of two ways: a) by replacing the observation with 1.0 so that ln (x_i) = 0 when x_i = 0; or, b) replacing the zero with very small values. For a more detailed discussion of zero-valued transformation, see MaCurdy and Pencavel (1986); Jacoby (1992); and Soloaga and Moss (2000). We recognize that these procedures are arbitrary and force the production function to include input quantities that are not actually observed.

The multi-objective artificial neural networks

ANNs use a connectionists approach to adaptive computation that respond to changes in data structure for information processing. In essence, ANNs represent an approach to non-linear statistical modeling. The objective of this paper is to estimate and compare the elasticity metrics obtained by applying a production-theoretic non-linear RANN to those obtained from applying the more traditional double-log OLS specification. In this regard, the topology known as the K4-RANN has proven robust in modeling financial data over several different time scales. For example, for hourly findings see (Dash *et al*. (2003)) and (Dash and Kajiji (2003)); and, Dash and Kajiji (2008)) for performance within the monthly domain. For comparisons across alternative ANN topologies, see (Dash and Kajiji (2002)). As with the generalized RANN method, the optimal weighting values w_j are extracted by applying a supervised least-squares method to a subset (training set) of the data series (Figure 1).

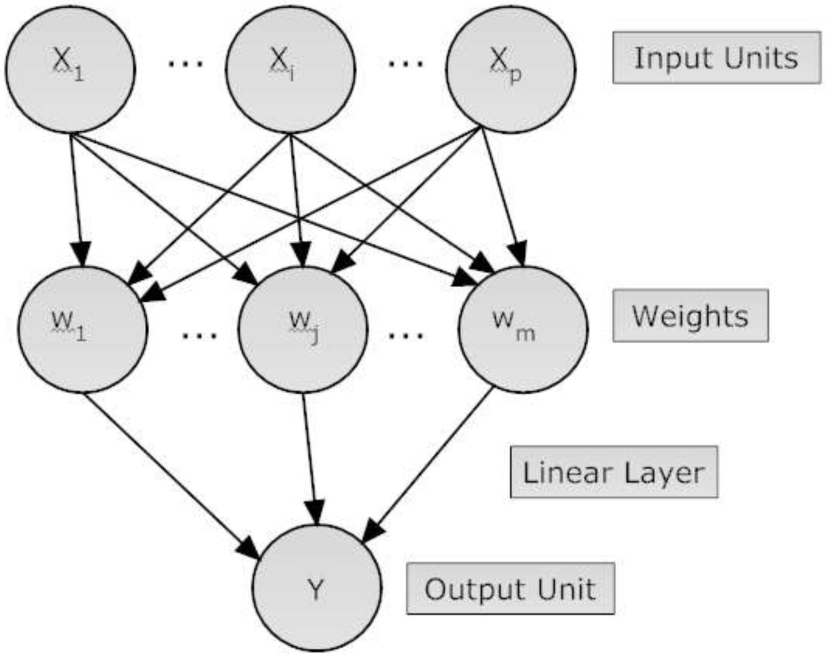

Fig. 1. K4-RBF Flowchart

The supervised learning function is stated as, $\boldsymbol{y} = f(\boldsymbol{x})$ where $\boldsymbol{y}$, the output vector, is a function of **x**, which is a *p by n* input matrix where *n* is the number of observations. The function can be restated as:

$$f(\mathbf{x}) = \sum_{j=1}^{m} w_j h_j(\mathbf{x}) \tag{3}$$

where, *m* is the number of basis functions (centers), ***h*** is the vector of hidden units, ***w*** is the weight vector. The augmented K4-RANN (Kajiji (2001)) becomes a multiple objective algorithm by adding a Tikhonov (1977) regularization function to the sum of squared errors (SSE). Presented differently, the K4-RANN minimizes a cost function that is defined as a modified SSE which is augmented to include the regularization parameters, ***k***. By definition, ***k*** is a weight decay parameter vector which penalizes mappings that are not smooth. Essentially, Tikhonov's regularization function provides additional information in order to mitigate any overfitting that may stem from an ill-posed problem:

$$\frac{\operatorname{argmin}}{k}\left(\varsigma\sum_{i=1}^{p}(y - f(x_i \mid \bar{k}))^2 + \sum_{j=1}^{m} k_j w_j^2\right) \tag{4}$$

Equation 4 expresses the objectives of mapping smoothness and modeling accuracy. Unfortunately, as first introduced by Hoerl and Kennard (1970) and Hemmerle (1975), and as popularized by Orr (1996; 1997), the solution to the weight decay parameter is often derived by computationally slow iterative techniques used in most non-linear least squares algorithms. The Bayesian-enhanced K4-RANN algorithm was designed to explicitly eliminate this computational bottleneck. The method derives a globally optimized regularization parameter based on a cross-population of recent Bayesian closed form extensions in optimal ridge regression offered by Crouse (1995). With this extension the design of the K4 algorithm is able to directly attack the twin evils that deter efficient ANN modeling: the "*curse*" of dimensionality and inflated residual sum of squares. The result is a set of weights (***w***) that minimize error (SSE) while optimizing the accuracy of the predicted fit (smoothness). In summary, the K4-RANN weights are analogous to nonlinear least squares regression parameters.

The data

Data is collected from two sources. These are, Rhode Island Department of Education (RIDE) and the National Center on Public Education and Social Policy (NCPE). Predictor variables from RIDE consisted of: a) academic year panel data for 2004-2005 on individual school demographics and assessments obtained for the population of 50 publically funded Rhode Island high-schools during the study period, and b) school related information such as, attendance; enrollment, poverty factors, etc. as guided by the *No Child Left Behind Act (2001)*.

Missing data was minimal and, when encountered, it was replaced by a bootstrap method. The total sum of missing data was described by two schools not reporting per pupil expenditures; one school that did not have an index score for the variable *instruction LSI;* and one school that did not report the *percentage of teachers with emergency certification*.

Predictor variables

NCPE-based predictor variables represent a subset from various *High Performance Learning Community Assessment: School Improvement Self-Study Survey* (1998, 1987). These surveys (HiPlaces) were administered to students, parents, teachers, and administrators of all public schools in Rhode Island. The survey (designed separately for each group) varied in size from a single page to 15 pages. The purpose of each survey was to ask specific questions that would enable school administrators and state legislators to construct a statistically sound school improvement plan dealing with all facets relevant to a child's education – environment, teaching practices, parents and community involvement, administration, health and social factors, etc. Every year the survey results were analyzed and a complete report was sent to the school.

At the conclusion of the initial data collection phase and just prior to the model formulation phase of the study a total of 33 variables were obtained from RIDE and, similarly, 21 variables were obtained from the HiPlaces survey. Variance and interaction effects were added to the dataset by the inclusion of 5 squared and cross product terms. The interested reader is referred to the author's data repository for details and downloads (http://www.nkd-group.com/research/jaor/exhibits-cb.pdf).

Data reduction

Owing to the large number of correlated variables obtained from the HiPlaces survey, policy segments (e.g., the health dimension and the teaching practices dimension) were factor analyzed using a Varimax rotation to reduce the variables to orthogonal domains. The factor procedures reduced the 12 health related variables to 2 orthogonal domains (Table 1). For convenience we refer to the factors as *Health 1* and *Health 2*, respectively. The 6 teaching practices variables all loaded on the first extracted factor. For subsequent econometric modeling purposes, predictor variable estimates were obtained by extracting factor scores for each factor domain using the maximum validity method of Thomson's (1951). In summary, 12 health dimension variables were replaced by two factor score variables and the 6 teaching practices variables were replaced by one unified factor score. The interested reader can access the details on the teaching variables from the data repository. It is not presented in this paper as the variable was not significant in the policy model.

Table 1. Sorted Varimax Rotated Factor Loadings for the Health Questions

Description	*Factor 1*	*Factor 2*	*Row %*
Used illegal drugs in last 30 days	0.939	.	0.883
5 or more alcoholic drinks once or twice each weekend	0.930	.	0.875
Drank alcohol at least once in last 30 days	0.892	.	0.795
Smoked marijuana regularly	0.864	.	0.759
Smoked at least one day in last 30 days	0.849	.	0.764
Problem with acne, overweight, underweight, etc.	0.748	.	0.563
Chewing tobacco or snuff in last 30 days	0.630	.	0.545
Watch TV 2 hours or more on an avg. school night	-0.623	0.577	0.720
On a typical night they get 7 hours of sleep or less	0.605	0.528	0.645
IM, Email, etc. 2 hours or more on an average school night	0.446	.	0.209
Did not eat fruits or vegetables in the past day	.	0.855	0.815
Ate breakfast 2 times or less in a week	.	0.850	0.742
Eigenvalues	*6.023*	*2.293*	
Cumulative percent of Explained Variance	*0.502*	*0.693*	

Target /dependent variables

The state of Rhode Island administers two standardized tests to all 11th grade students; the Math and English Language Arts tests. Each test dimension has several sub-tests that are averaged over a three year period to create the dependent variables for this study. For this study the target, or dependent, variables are computed as a weighted average of individual scores for the academic years 2003, 2004, and 2005. The two dependent variables are referred to as the *Three-Year Math Index Score* (MAI) and the *Three-Year English Language Index Score* (ELAI).

Modeling results

Generalizing the double-log production function

The double-log specification of the MAI and ELAI for the included variables is as follows:

$$Ln(MAI) = Ln(\alpha) + \sum_{i=1}^{p} \beta_i Ln(x_i) + \varepsilon \text{; where } \varepsilon \sim N(0,1). \quad (5)$$

$$Ln(ELAI) = Ln(\alpha) + \sum_{i=1}^{p} \beta_i Ln(x_i) + \varepsilon \text{; where } \varepsilon \sim N(0,1). \quad (6)$$

The above specification of the models was applied to the population of 50 high schools using traditional OLS procedures (where: p is the number of predictor variables and will be different for each model). The final MAI model includes 18 statistically significant explanatory variables and the ELAI model has 7 explanatory effects. The MAI model solves with an overall F-statistic of 436.337 which is significant at the 99-percent confidence level. The adjusted R-square for the model is 99.379 percent with an average VIF of 5.045. Although the VIF value is just slightly greater than 5.0, it is close enough to this watershed value that ills associated with multicollinearity are not present in this solution. Upon review of diagnostic graphical support it is evident that the MAI model produced residuals that are characteristic of constant variance and a normal distribution. The graphical analysis was supported by the statistic on the *correlation of normality* which has a computed value of 0.994.

The ELAI model shows similar results. We have an overall F-value of 1025.864 which is significant at the 99-percentile. The adjusted R-square is 99.322% with a very small variance inflation factor of 3.596. The *correlation for normality* statistic of 0.995 suggests that the residuals are normally distributed.

Except for two variables of the MAI model, *% Teachers with Emergency Certifications* and *PPE substitutes*, all parameter estimates for both econometric models were significant at the 99% level. The two not meeting this threshold were significant at the 95% level. The individual VIFs for all variables across both models were below the threshold of 10. This, finding indicates an absence of severe multicollinearity in the models. However, we did observe that 50 percent of the VIFs in the MAI model and 14 percent in the ELAI model measured in the 5 to 8 range. Contemporary views as summarized by O'Brien (2007) suggests the application of a ridge-regression to uncover consistent estimates.

The K4-RANN

Although the number of observations available for this study is moderate, a number of studies have found that RANNs perform well on small data sets. Simon (2002) implemented as few as 25 exemplars in the training phase. Aydin (2009) used as few as 24 observations in the validation phase in the time series study of Turkish tourism. Idri, *et al*. (2010) conducted several experiments with as few as 32 hidden neurons.

The two estimated models are stated as,

$$Ln(MAI) = \sum_{i=1}^{p} \beta_i Ln(x_i) + \varepsilon \tag{7}$$

$$Ln(ELAI) = \sum_{i=1}^{p} \beta_i Ln(x_i) + \varepsilon \quad . \tag{8}$$

The model building process tested several K4-RANN formulations where each used a different data and scale transformation of the target and independent variables. In each case, supervised training for any particular K4 RANN model used 42 percent of the data points (21 observations or schools). Each estimated model was subjected to validation by using all remaining data observations (29 observations). All RANN estimations relied upon the *Generalized Cross Validation* (GCV) error minimization rule introduced by Golub *et al*. (1979). GCV requires an adjustment to the average mean-squared error over the training set. Upon producing alternative estimated models for both MAI and ELAI specifications, the 'best" predictive policy model was determined based on the overall quality of the following performance metrics: MAPE, AIC, BIC, and the Thiel *"U"* statistic. We relied specifically on the smallest value rule (largest negativity rule) for AIC and BIC to provide a comparative measure for model selection. The Thiel "*U*" statistic was used to assess whether the forecast results obtained were better than those attributable to a naïve or trivial prediction. An estimated model with a Thiel "*U*" close to zero is preferred.

For the MAI model, the K4-RANN results show an R-square value of 99.96% and a MAPE of 0.3343. The AIC, BIC, and the Theil's *"U"* measures are reported as -346.977, -312.561 and 0.00258, respectively. The ELA model produced similar results. The R-square value for this model is slightly lower with a recording of 99.03%. The model also solved with a slightly higher MAPE of 0.51, with AIC and BIC values of -322.259 and -318.875, respectively. The Theil's *"U"* is reported at zero. The estimated policy parameter weights generated by the selected models are presented and discussed in the following sections.

Policy implications

In order to enhance comparative analytics, each policy dimension is presented below with the supporting parameter or weight estimate produced for that dimension. Where a dimension is grouped to form a global dimension, the global dimension elasticity is also presented. We note that the double-log parameter values are direct estimates of a dimension's elasticity with respect to MAI and ELAI. This is not strictly true for the estimated weights of the RANN. As argued above, the K4-RANN weights are analogous to parameter estimates generated by a nonlinear regression. For the purposes of this paper we shall treat the RBF weights estimated by application to the doublelogarithmically transformed data as a 'quasi' elasticity estimates. This assumption is made for comparative analysis only. The analysis of each policy dimension follows.

Non-white, and interaction of non-white with free and reduced lunch

Table 2. Policy Dimension 1

	Math Achievement Index	
	Double-Log	*K4-RANN*
Non-White	-0.012	0.036
Free-Reduced Lunch with Non-White	0.006	-0.160
Sub-dimension Elasticity	-0.006	-0.124

The sub-dimension elasticity measure in Table 2 is negative for both the DL and RANN models. Interestingly, the domain elasticity coefficients experience reversed signs for the individual dimensions. The New England experience with racial diversity in public schools has a deeply documented history (see, Edwards and Willie (1999) for a review). The more contemporary argument has been that diversity goals alone can lead to positive interactions between different racial / ethnic groups (Civil Rights Project, (2002)). Recent practice suggests that ability grouping is correlated with both mathematics achievement and racial diversity and thus becomes a reason some schools continue to employ ability grouping as a means by which to segregate students (Vanderhart, (2006)). While this research does not delve into the issue of ability grouping we do find evidence that could serve as grounds for justifying the use of racial-based achievement as a reason to support such a segregationist policy. Under the DL model, we find evidence that MAI will decline as the percentage of non-white students in a school increases. The RANN generated solution of 3.6 reports exactly the opposite effect – a potential for a 0.036 percent increase in MAI given a 10 percent increase in the non-white high school population. However, the RBF findings also suggest that as the percentage of the non-white population that requires a free or reduced cost lunch increases, there is a reduction in the MAI scoring performance by approximately 12.4 percent. The combined effect of these two sub-dimensions clearly suggests that MAI performance is not affected solely by any arbitrary increase in racial diversity. Instead, it is an increasing non-white student population from families with an income stratum that causes acceptance into the free and reduced cost lunch program that leads to the downward influence on overall MAI scoring.

Parent graduated from high school

Table 3. Policy Dimension 2

	Math Achievement Index	
	Double-Log	*K4-RANN*
Parent Graduated from High School	-0.025	0.062

The two computational methods produce reversed signs within this dimension (Table 3). Tracing family contribution back a generation once parents' characteristics are held constant there is little evidence of significant independent effects of grandparents' social or economic statuses on the educational attainments of grandchildren (Warren, *et al*. (1997)). However, when family wealth is considered the inter-generational effect on literacy takes on a slightly different view. There is significant evidence that wealth is correlated with social and economic standing in the parental generations; a finding that suggests inter-generational family wealth may influence current generation literacy rates. Given the findings provided by the National Center for Education Statistics (2006), on average high school completers experienced higher income than non-completers and early completers experienced greater wealth accumulation than late completers. These results provide support for the positive elasticity weight of 0.062 for the RANN production model. By contrast, the DL model extracts a small negative relationship which is not quite as plausible.

High School Attendance and Suspensions

Table 4. Policy Dimension 3

	Math Achievement Index		*ELA Achievement Index*	
	Double-Log	*K4-RANN*	*Double-Log*	*K4-RANN*
Attendance – High School	1.243	0.007	0.155	0.175
Suspensions – High School	0.012	-0.042	0.005	0.197
Sub-dimension Elasticity	1.255	-0.035	0.155	0.372

We view high-school attendance and suspensions to be linked in their relationship to literacy and attainment. Tracing a historical review back to 1910, Fishback and Baskin (1991) argue that attendance is one of several important factors that impact the black-white gap in literacy. The question then is, "What deters increasing stable attendance rates at the high school level?" One known deterrent is school violence (Jackson, (1994)). When a high school is known to be disorderly it is important for administrators to adhere to their educational mission in a consistent manner. This may include suspending offending students. The total sub-dimension elasticity shows a large conflict in potential policy inference between DL findings and RANN findings (Table 4). The DL findings produce a positive elasticity for both dimensions (attendance and suspensions). The elasticity for attendance suggests that for every 10 percent increase in attendance rates, MAI will increase by more than 12.4 percent. Under DL findings, the improvement in MAI rates is further buttressed by increased incidents (not individual students) of suspensions. By contrast, the RANN findings offer a different and more conservative view. Attendance relates positively to MAI but with a much lower and when all other effects are held constant, the expected increase of 0.07% in MAI for a 10 percent increase in attendance. The quasi-elasticity for suspensions under the RANN solution is inversely related to MAI. This would suggest that when school order is maintained by increasing the number of suspension incidents there is an overall dampening effect on MAI. Unlike the MAI model, the ELAI model shows a positive weight for both the DL and RANN. Essentially the ELAI model indicates that these two variables, attendance and suspensions, have the maximum impact on increasing overall achievement performance.

% of Teachers with Emergency Certification

Table 5. Policy Dimension 4

	Math Achievement Index	
	Double Log	*K4-RANN*
% Teachers with Emergency Certification	0.002	0.013

Stylized facts needed to explain the variation in state education policies (teacher certification) and the effect on student performance is mixed. Berger and Toma (1994) find that students in states with a master's degree requirement for teacher certification had lower SAT scores than did students in states without a master's requirement. For the RI high-school population state regulations require all teachers in the public schools to hold a state certification. However, infrequently some teachers require emergency certification. For the population under study the permissibility of employing teachers who are temporarily certified does lead to a positive effect on the MAI scores (Table 5).

Math v/s ELA Achievement Index (AY 2002-04)

Table 6. Policy Dimension 5

	Math Achievement Index		*ELA Achievement Index*	
	Double-Log	*K4-RANN*	*Double-Log*	*K4-RANN*
MAI (AY 2002-2004)	0.735	0.031		
ELAI (AY 2002-2004)			0.793	0.220

Smith and Schumacher (2006) provide important evidence that links prior performance in mathematics with future success in applied disciplines that rely upon fundamental mathematics training. The dimension included in this study is the MAI for the academic year 2003-04 or the ELAI score for the respective models. The positive and high elasticity (Table 6) corroborates the implications provided by earlier studies. Also, most schools do not undergo any form of major restructuring in the school population thus confirming our hypothesis that the prior year index score is a significant predictor of the current year MAI or the ELAI score.

Per Pupil Expenditures

Table 7. Policy Dimension 6

	Math Achievement Index		*ELA Achievement Index*	
	Double-Log	*K4-RANN*	*Double-Log*	*K4-RANN*
PPE – Classroom Technology	0.017	0.009		
PPE – Classroom Materials	0.019	0.097	0.014	0.062
PPE – Classroom Teachers	-0.111	-0.058		
PPE – Substitute Teachers	0.006	-0.141		
PPE – Operations	0.062	-0.001		
Sub-dimension Elasticity	-0.007	-0.094		

The results presented in Table 7 will surely displease some school administrators. The sub-dimension elasticity under both DL and RANN models is negative. On average, these findings mean that increasing the per-pupil expenditure (PPE) will not lead to positive changes in MAI scoring. Peering into the actual PPE dimensions yields important policy-related insight such as in the case of substitute teachers. The same reversal of signs is observed for PPE of Operations. Of these two, the RANN finding would clearly indicate that an overuse of substitute teachers will result in a decline of MAI. Further, simply spending additional funds on academic operations will also fail to generate desired outcomes in MAI. Both models find that policy attention should be directed at increasing teaching materials and classroom technology. Interestingly for the ELAI model the most significant variable in terms of the PPE is the classroom materials. All other PPE were not significant. Resources for student teaching in general are the most significant expenditure one needs to undertake to raise overall achievement scores.

Learning Support Indicators

In December of 2002, RIDE and NCPE together developed a set of five school characteristics as important benchmarks for educators and policy makers to focus on as they develop a school improvement plan. These five characteristics for the high schools are: *Instruction, Parent Involvement, School Climate, Attendance*, and *Dropout Rate*. For details on the construction and development of each LSI see the Technical bulletin prepared by McWalters, *et al*. (2008).

Table 8. Policy Dimension 7

	Math Achievement Index		*ELA Achievement Index*	
	Double-Log	*K4-RANN*	*Double-Log*	*K4-RANN*
LSI – Instruction	-0.064	-0.054		
LSI – Parent Involvement	0.062	-0.097		
LSI – School Climate			0.107	-0.035

Instruction LSI

The empirically estimated elasticity for the Instruction LSI for MAI is -0.064 and -0.054 as given by the DL and RANN models, respectively (Table 8). These findings indicate that there is a negative impact on MAI. Upon investigation of the Instruction LSI domain it is clear that at the State high school level *Teacher Preparation in Standards-based Instruction* is measurably impaired with a score of 28 (see Data Repository). Additionally, despite best efforts, the low score of 24 suggests that current teaching practices do not implement *Integrated Thematic Units in Reading and Mathematics*. The negative interaction in the model is further explained by the fact that while high-school students have the benefit of the teachers practicing *Classroom and Cross-Teacher Grade Level Practices* the mid-level score recorded for *Barrier to Implementation of New Curriculum Based Instruction* is counterproductive to support learning and achievement. Unless administrators prepare teachers to deliver standards-based instruction, it is unlikely that RI high schools will report an increase in MAI.

Parent Involvement LSI

Children's literacy from elementary school to high school is affected by parental involvement. Dearing, *et al*. (2006) find that increased parental involvement between kindergarten and 5^{th} grade is associated with increased literacy performance. Gonzalez and Wolters (2006) focused on motivational attitudes and find that authoritarian parenting was positively related to performance in mathematics. As shown in the Data Repository the dimension for the Parent Involvement LSI focuses on the interaction of the endogenous effects of teaching with the exogenous involvement of parents in four specific communication areas. The sub-dimension, *Teacher Reports of Parent Contact Regarding Schoolwork and Homework,* shows an index score of 68 where as *Teacher Reports of Contact with Parents* achieves a value of 72. The index scores indicate that teachers only focus on two of the four dimensions. These two dimensions focus on the principle of 'direct contact' *with parents* as it pertains only to the child's performance in school.

The DL model reports an elasticity of 0.062, or we may expect on average of 6.2 percent change in MAI with a modest 10-percent increase in the contact between parents and teachers. The RBF model contradicts with a value of -0.097 suggesting that a decline of 9.7 percent in MAI is to be expected with each 10 percent increase in parental involvement. These contradictory findings suggest that policymakers need to carefully construct the framework by which to increase parent involvement. For example, if parents' have not graduated from high school but attempt to increase their involvement in the child's mathematics learning process, the resultant outcome may not lead to the desired increase in MAI. Setting aside the discrepancy of the estimated elasticity coefficients, these findings are consistent with contemporary findings as they relate to the parent involvement and academic literacy. Stylized facts continue to argue for policymakers to augment their efforts at installing programs that encourage teachers and parents to seek a broader common ground of achievement-based communication.

School Climate LSI

The school climate indicator did not play a significant role in explaining the variation in MAI. It does however play a significant role in explaining the variation in ELAI. Once again the results from the DL and RANN models are in conflict. The DL model indicates that there would be a significant increase (10.7 percent increase for every 10 percent change in the school climate) the RANN model on the other hand indicates a slight decrease of 3.5%. Effective school reform ultimately leads to improvement in students' academic performance. The environment that affects the behavior of teachers and students is captured in their shared beliefs and attitudes that characterize the district-wide organization and establish boundaries for all constituents. Specifically, the S*chool Climate LSI* is characterized primarily by the variable *Student Expectations* which has a score of 83 (see Data Repository). The other variable that makes a substantial contribution to the index is *Classroom Climate* with a score of 70. Climate is a "feeling" that can vary at the school level. On the whole when students feel safe in a congenial classroom and know what is expected of them; their performance in ELAI seems to improve.

Health Factor 2

Table 9. Policy Dimension 8

	ELA Achievement Index	
	Double Log	*K4-RANN*
Health Factor 2	-0.018	0.002

Health –related factors are known to have a direct impact on academic factors. Hunger, physical education, emotional abuse and chronic illness are known to dampen overall school performance. Recent research has found that, after controlling for sex, race/ethnicity, and grade level, there is a negative association between health-risk behaviors and academic achievement among high school students (e.g., see Murray, (2007)). As indicated by the negative -1.8% for the DL model and only a very small positive elasticity of 0.2% for the RANN the *health factor 2* suggests an overall deterioration in the ELAI scores for every 10% increase in *health factor 2* (Table 9). Upon investigation of the composition of *health factor 2* (Table1), it is evident that lack of a proper breakfast, increased TV watching, and lack of sleep are the reasons this factor has a negative impact on ELAI scores.

Indicator Variables – Urban Code, Urban Ring, and Program

Table 10. Policy Dimension 9

	Math Achievement Index		*ELA Achievement Index*	
	Double-Log	*K4-RANN*	*Double-Log*	*K4-RANN*
Urban Code	0.082	-0.314		
Urban Ring	0.039	0.078		
Program	0.268	0.095	-0.066	0.175

Three indicator variables were included in each model specification. *Urban Code* indicates which schools were classified as belonging to an urban district. The *Urban Ring* indicator identifies schools that surround, but are not part of, an urban district. For a school that supports a special program (e.g., learning support for the hearing impaired) the *Program* indicator variable's default value of zero is replaced by the value of one (1.0). The *Urban Code* and *Urban Ring* are both specified in the MAI model (neither is specified in the ELAI model) and both are statistically significant. Program code is estimated in both models and is significant in each policy model. Although MAI is impacted positively for schools in the urban ring, there is a conflict in the findings for those schools classified as urban. Under the K4-RANN, Table 10, the effect is negative and sizeable (-0.314). This is direct contradiction to the positive and decidedly smaller elasticity reported under the double-log specification. Conventional wisdom suggests that urban schools continually fail to reach threshold levels in the study of the mathematics. For this set of findings, the nonlinear mapping of the K4-RANN clearly appears to be more consistent with contemporary thought.

The conflicting findings for the *Program* indicator variable in the ELAI model also seem to favor the results generated by the K4-RANN model. Under the double-log findings, any increase in special programs would produce a decline in the school's aggregate measure of ELAI achievement. However, to accept this finding is tantamount to suggesting that special programs fail at their appointed task. The positive quasi-elasticity of 0.175 generated by the K4-RANN estimation is far more plausible given the stated objectives of a directed education program.

Summary and Conclusions

Educational attainment is a necessary ingredient for any society to experience the benefits of economic, cultural and social growth. In this paper we examined alternate modeling structures to estimate production-theoretic elasticity metrics for high-school mathematics and English arts achievement in the state of Rhode Island. We find that for a policy dimension like *% of Teachers with Emergency Certification*, the traditional double-log model and the nonlinear K4-RANN model produced similar elasticity metrics. For other dimensions, such as Attendance and Suspensions, the alternate modeling results were contradictory. However, experienced interpretation favored the scale implications extracted from the K4-RANN model. The K4-RANN results are also favored due to the implied ridge-regression methodology that is embedded in the algorithm. The empirical evidence produced by the nonlinear RANN model clearly argues for continued research in artificial intelligence to extract efficient predictors of educational performance and resultant policy inference in Rhode Island. Note: *The University of Rhode Island, closed NCPE in 2009 and archived all proprietary internet data.*

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